MATH348. Harmonic Analysis. Problems 2

Work due on *Wednesday 13th October.*

1. Let $f : (-\pi, \pi] \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} 1 \text{ if } 0 \leq x \leq \pi \\ 0 \text{ if } -\pi < x < 0 \end{cases}$$

Extend to a 2π-periodic functions, and call this f also. a) Give the values of

$$\frac{f(0+) + f(0-)}{2}, \quad \frac{f(\pi+) + f(\pi-)}{2}, \quad \frac{f((\pi/2)+) + f(\pi/2)-)}{2}.$$

b) Compute the Fourier coefficients $\hat{f}(n)$. Use the pointwise Fourier Series Theorem at $\pi/2$ to show that

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}.$$

c) Use Parseval's equality applied to f to show that

$$\sum_{n=0}^{n=\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}.$$

2. Determine which of the following functions are integrable.

a) $f(x) = x^{-3/4}$ on $(0, 2\pi)$.

b) $f(x) = x^{-4/3}$ on $(1, \infty)$.

c) $f(x) = x^{-3/4}$ on $(0, \infty)$.

d) $f(x) = x^{-4/3}$ on $(0, \infty)$.

e) $f(x) = (\sin^3 x)x^{-3}$ on $(0, \infty)$.

3. Determine which of the functions in question 2 are in (i) L^1, (ii) L^2, (iii) L^∞.