Conformal Mappings by Elementary Holomorphic Functions

MATH206 Project A and C (after MATH243)

Conformal mappings are those preserving angles: if two curves meet at some angle then their images under such a mapping meet at the same angle. Holomorphic functions with non-zero derivatives are the major example of this kind.
Apart from its importance in Pure Mathematics (complex analysis, topology, geometry, etc.) and modern Theoretical Physics (e.g. conformal field theory), the elegant theory of conformal mappings has numerous applications in Applied Mathematics (solving various partial differential equations).
The projects will concentrate on the study of the actions of the elementary holomorphic functions on standard curves and regions on the complex plane. The functions are mostly those well-known from the M243 course: exponential, power, trigonometric, hyperbolic, and their compositions. Curves are mainly co-ordinate curves of the Cartesian and polar co-ordinate systems. The most popular region in the unit disc $|z| < 1$: according to a striking theorem of Riemann, any other region of the plane can be mapped conformally and one-to-one to the disc provided the region has no holes and is not the entire plane.
To get a taste of the subject, open Priestley's textbook and briefly look through Sections 10.15–10.20 in it.
The project is for a group of (up to) 8 students. All the theory for all the members is the same; however, theory is divided into pieces, a piece for each member of the group to write it up and be responsible for at the oral presentations and during the whole group work over the project. The problems are all individual.

Contents:
Complex differentiation and the Cauchy-Riemann equations.
Geometric interpretation of the derivative. Conformal mappings.
Functions z^n and e^z.
Fractional-linear functions. Function $(z + z^{-1})/2$.
Trigonometric and hyperbolic functions.
Constructing conformal mappings. The Riemann mapping theorem.
Optional: Multi-valued functions: roots, logarithms, $\cos^{-1} z$.

Sources (the overlap is non-empty):
H. A. Priestley, Introduction to Complex Analysis, Clarendon Press, Oxford, 1995.
(Copies of the relevant pages can be requested)
A. I. Markushevich, Theory of Functions of a Complex Variable, vol.I, Prentice-Hall, 1965.
(Copies of the pages needed will be provided)