MATH191: Problem Sheet 10

1. Write each of the following series using the $\sum$ notation. The ratio test shows that one of them is convergent, and one is divergent; the convergence or divergence of the third cannot be determined using this test. Which is which?

a)

$$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{r^2} + \cdots$$

b)

$$\frac{1}{0!} + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots + \frac{2^r}{r!} + \cdots$$

c)

$$\frac{1!}{3} + \frac{2!}{3^2} + \frac{3!}{3^3} + \frac{4!}{3^4} + \cdots + \frac{r!}{3^r} + \cdots$$

2. Calculate the radius of convergence R of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} x^n.$$

Write down the series when $x = R$ and explain why it converges by using the alternating series test. Write down the series when $x = -R$, and explain why it diverges. Hence state all of the (real) values of x for which the power series is convergent.

3. Calculate the radius of convergence R of the power series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 3^n} x^n.$$

Write down the series when $x = R$ and when $x = -R$, and explain why it is divergent in each case. Hence state all of the (real) values of x for which the power series is convergent.