MATH191: Practice Sheet 5

1. Differentiate the following functions. There is no need to simplify your answers.

a) $2x^3 - 5x + 1 - 2\sin x$; b) $x^{5/3}$; c) $(1 - x^2)\tan x$; d) $\sqrt{2x+3}\cos x$;

e) $\dfrac{x^2 - 3}{2x}$; f) $\dfrac{x^2 \cos x}{\sqrt{x} - 1}$; g) $\cos(-3x + 2)$; h) $(1 - x^2)^{-3/2}$;

i) $\dfrac{\sin(2x - 1)}{x}$; j) $\dfrac{\cos((1 - 2x)^2)}{3x + 1}$.

2. Let $f(x) = x^2 - 5$ (so a solution to $f(x) = 0$ is $x = \sqrt{5}$). Apply the Newton-Raphson method to $f(x)$, with a starting value $x_0 = 2$, to compute $f(x_n)$ and x_n for $0 \leq n \leq 3$. You should give each approximation to 6 decimal places. Check your answer by evaluating $\sqrt{5}$ on your calculator.

3. By sketching the graphs of $y = x^3$ and $y = 1 - x$ on the same axes, explain why the equation

$$f(x) = x^3 + x - 1 = 0$$

has exactly one solution which is in $(0, 1)$

Use the Newton-Raphson formula with an initial guess $x_0 = 1$ to to compute $f(x_n)$ and x_n for $0 \leq n \leq 3$. You should give each approximation to 6 decimal places.