

MATH191: Practice Sheet 10

These problems test the material in the final section of the course. You need not hand them in (as there will not be time for them to be marked). Solutions will be given out next week but you should attempt the questions first!

1. Write each of the following series using the \sum notation. The ratio test shows that one of them is convergent, and one is divergent; the convergence or divergence of the third cannot be determined using this test. Which is which?

a)

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{r^2} + \cdots$$

b)

$$\frac{1}{0!} + \frac{3}{1!} + \frac{5}{2!} + \frac{7}{3!} + \cdots + \frac{2r+1}{r!} + \cdots$$

c)

$$\frac{2}{1} + \frac{4}{8} + \frac{8}{27} + \frac{16}{64} + \cdots + \frac{2^r}{r^3} + \cdots$$

2. Calculate the radius of convergence R of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n.$$

Write down the series when $x = R$ and explain why it converges by using the alternating series test. Write down the series when $x = -R$, and explain why it diverges. Hence state all of the (real) values of x for which the power series is convergent.

3. Calculate the radius of convergence R of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}4^n} x^n.$$

Write down the series when $x = R$ and when $x = -R$, and explain why it is convergent in each case. Hence state all of the (real) values of x for which the power series is convergent.

Final course arrangements and notes on revision

I hope to finish the course material by the end of the lecture on Monday 13th December at the latest. This final material is important and could be tested in the exam. I shall use the remaining lectures to go over past exam questions. I strongly recommend that you attend these sessions.

In your revision, it is a good idea to tackle past exam papers — which are available on the module webpage — *but also* to go over the problem sheets and practice problem sheets. The problems in the exam will be similar to the problems you have tackled during the term *as well as* being similar to past exam problems. When you come to the exam, read the questions on the paper in front of you. Be careful not to confuse them with questions on previous exams, or any questions that you have previously done. They may be similar *but not identical*.