## SECTION A

**1.** State the maximal domain and range of the function

$$f(x) = \ln(x - 2).$$

Sketch the graph of $y = f(x)$, indicating the coordinates of any points where the graph crosses the axes.

[4 marks]

**2.** By evaluating $f(0)$, $f'(0)$, and $f''(0)$, obtain the Maclaurin series expansion of the function

$$f(x) = \frac{1}{\sqrt{x+4}}$$

up to and including the term in $x^2$.

[5 marks]

**3.**

a) Convert $(x, y) = (-1, 1)$ from Cartesian to polar coordinates.

b) Convert $(r, \theta) = (2, \pi)$ from polar to Cartesian coordinates.

In both parts, full marks will only be obtained if exact answers are given in terms of $\pi$, $\sqrt{2}$ etc..

[5 marks]

**4.** Evaluate the definite integral

$$\int_1^2 \left( e^{2x} + \frac{1}{\sqrt{x}} \right) \mathrm{d}x,$$

giving your answer exactly in terms of $e$ and $\sqrt{2}$.

[5 marks]

**5.** Consider the curve defined by

$$x^3 + x^2y + xy^2 = 1.$$

Find an expression for $dy/dx$ in terms of $x$ and $y$, and hence give the equation of the tangent to the curve at the point $(x, y) = (1, -1)$.

[8 marks]

**6.** Differentiate the following functions with respect to $x$:

a) $x^2 \cos 2x$; b) $(x^2 + x + 1)^9$; c) $\dfrac{e^x}{x^2 - 1}$.

[7 marks]

**7.** Show that the function

$$f(x) = e^{-x} + x$$

has exactly one stationary point. Determine whether the stationary point is a local maximum, a local minimum, or a point of inflection.

[5 marks]

**8.** Let $z_1$ and $z_2$ be the complex numbers given by $z_1 = 2 + j$ and $z_2 = 3 - j$. Calculate $z_1 + z_2$, $z_1 - z_2$, $z_1 z_2$, and $z_1/z_2$.

[6 marks]

**9.** State the value of $\sin^{-1}(-\frac{1}{2})$ (you should give an exact answer in radians, in terms of $\pi$). Give the general solution of the equation

$$\sin\theta = -\frac{1}{2}.$$

[4 marks]

**10.** Let $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Find $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $|\mathbf{a}|$, $|\mathbf{b}|$, and $\mathbf{a} \cdot \mathbf{b}$. What is the angle between $\mathbf{a}$ and $\mathbf{b}$?

[6 marks]

SECTION B

**11.** Give the Maclaurin series expansion of the function $f(x) = e^x$ up to and including the term in $x^4$. (You are not required to show any working if you remember this expansion.)

[2 marks]

Hence, or otherwise, determine the Maclaurin series expansions of the following functions, up to and including the terms in $x^4$:

a) $e^{2x}$; b) $e^{-x}$; c) $e^x + e^{-x}$; d) $e^x - e^{-x}$.

[10 marks]

Use some of these Maclaurin series expansions up to the term in $x^4$ to obtain an approximation to $f(0.1)$, where

$$f(x) = \frac{e^{2x} + 1 - 2e^x}{x^2}.$$

You should give your approximation to 3 decimal places.

[3 marks]

**12.** In each of the following cases, calculate the radius of convergence $R$ of each of the power series, and determine whether the series converges at $x = \pm R$ in a), and at $x = R$ in b).

a) $\sum_{n=1}^{\infty} \frac{1}{2^n n^2} x^n$; b) $\sum_{n=1}^{\infty} \frac{1}{2^n \sqrt{n}} x^n$.

[15 marks]

**13.** Use calculus to show that the function $f(x) = x^3 + 3x^2 - 1$ has exactly three zeros, one in $(-3, -2)$, one in $(-1, -1/2)$ and one in $(0, 1)$.

[6 marks]

Use the Newton-Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

with an initial guess $x_0 = \frac{1}{2}$ to obtain successive approximations $x_1$ and $x_2$ to the positive solution of the equation $f(x) = 0$. Give $x_1$ and $x_2$ as fractions, and also to 6 decimal places. Also, give $f(x_2)$ to 6 decimal places.

[9 marks]

**14.** Let $f(x)$ be defined by

$$f(x) = \begin{cases} 1/(1+x) & \text{if } x \in (-\infty, -1) \\ 1 + (x/2) & \text{if } x \in [-1, 1] \\ (4 - x^2)/(1+x) & \text{if } x \in (1, \infty) \end{cases}$$

Find any horizontal and vertical asymptotes of $f$. Identify any points of discontinuity of $f$. Calculate $f'(x)$ where it exists. Identify any points where $f$ is not differentiable. Show that $f$ has no stationary points, and determine maximal intervals on which $f$ is decreasing and maximal intervals on which $f$ is increasing. Find any zeros of $f$. Sketch the graph of $f$.

[15 marks]

**15.** Find all $z$ in the form $a + bj$, for real $a$ and $b$, such that

a) $z^2 = -4j$; b) $z^3 = j$.

*Hint*: You might find de Moivre's Theorem useful. [15 marks]