1. Give the names of the following (lower case) Greek letters: λ, π. Write the lower case Greek letters *alpha* and *mu*. [4 marks]

2. For each of the following sets S, give a function $f(n)$ such that

$$S = \{f(n) \mid n \in \mathbb{N}\}.$$

(Note that 0 is considered to be a natural number.)

(a) $S = \{7, 9, 11, 13, 15, 17, \ldots\}$.

(b) $S = \{10, 100, 1000, 10000, \ldots\}$.

(c) $S = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$. [12 marks]

3. Negate each of the following statements:

(a) $x^2 \geq 1$.

(b) $x < 0$ or $x \geq 1$.

(c) If $x > y$ then $f(x) < f(y)$.

(d) $\exists M \in \mathbb{N}$, $\forall x \in \mathbb{R}$, $f(x) \leq M$. [12 marks]

4.

Definition: Let f be a (real-valued) function. Then f is *injective* if for all x, y in the domain of f,

$$f(x) = f(y) \implies x = y.$$

Working directly from this definition, determine whether or not the following functions are injective. You should justify your answers.

(a) $f(x) = 5 - 6x$.

(b) $f(x) = x^2 + 1$.

(c) $f(x) = x^2 + 1, x \geq 0$. [14 marks]

5.

Definition: Let S be a subset of $\mathbb{Z}$. Then S is *closed under addition* if for all $m, n \in S$, $m + n \in S$.

State carefully what it means for a subset S of $\mathbb{Z}$ *not* to be closed under addition.

Determine whether or not the following subsets S of $\mathbb{Z}$ are closed under addition. You should justify your answers from the definitions.

(a) $S = \{3k \mid k \in \mathbb{Z}\}$.

(b) $S = \{k \in \mathbb{Z} \mid k \leq 50\}$.

(c) $S = \{k \in \mathbb{Z} \mid k \geq 50\}$. [14 marks]

6. Consider the following theorem. You are not expected to understand what it means.

Theorem *Let X be a metric space. If X is complete and X is totally bounded, then X is compact.*

Identify the context, hypothesis, and conclusion of the theorem. State its contrapositive.

What, if anything, does the theorem tell you about a metric space X which is:

(a) Compact?

(b) Not Compact?

(c) Complete but not compact?

(d) Complete but not totally bounded? [14 marks]

7. Write proofs of the following statements. You should work from the definition:

Definition: Let $m, n \in \mathbb{Z}$. Then m *divides* n, written $m|n$, if there exists an integer k such that $n = km$.

(a) Let $m, n \in \mathbb{Z}$. If $7|m$ and $7|n$ then $7|(m + n)$.

(b) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.

(c) Let $m, n \in \mathbb{Z}$. If 7 divides m and 7 does not divide $m + n$ then 7 does not divide n. [15 marks]

8. Determine whether each of the following statements is true or false. Justify your answers briefly.

(a) $\exists n \in \mathbb{Z},\, n^2 = 5.$

(b) $\forall n \in \mathbb{Z},\, n^2 > 0.$

(c) $\forall x \in \mathbb{R},\, \exists y \in \mathbb{R},\, 3y = x.$

(d) $\forall x \in \mathbb{R},\, \exists y \in \mathbb{R},\, \cos y = x.$

(e) $\exists x \in \mathbb{R},\, \forall y \in \mathbb{R},\, -1 < x - y < 1.$ [15 marks]