Practice Class Test MATH102 2007

1. Write down the Taylor series at 0 of $f(x) = \sin 2x$.

[3 marks]

2. Write down the quadratic Taylor poynmial $P_2(x) = P_2(x, 4)$ near $x = 4$ for $f(x) = x^{1/2}$, and work out $P_2(3)$. Write down an expression for the remainder term $R_2(3)$. Now find an upper bound on $|R_2(3)|$ and hence show that

$$|\sqrt{3} - P_2(3)| \leq \frac{1}{16 \times 3^{5/2}} < 0.005.$$

Confirm this by using your calculator to compute $\sqrt{3}$.

[9 marks]

3. Solve

$$x\frac{dy}{dx} + y^2 = 0, \quad y(1) = 2.$$

[5 marks]

4. Solve

$$x\frac{dy}{dx} - y = x^2, \quad y(2) = 1.$$

[7 marks]

5. Find the general solution to

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0.$$

[4 marks]

6. Find the general solution to

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x}.$$

[6 marks]

7. Show that

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + 2y^2}{x^2 + y^2}$$

does not exist, by calculating limits along two different directions at $(0, 0)$.

[6 marks]