



THE UNIVERSITY
of LIVERPOOL

Department of Mathematical Sciences

MATH266

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Numerical Analysis: Solution of Linear Equations

Syllabus

- 1 — Solution of Non-linear Equations Approximation
- 2 — Solution of Linear System of Equations (direct).....Matrix
- 3 — Solution of Linear System of Equations (iterative) Matrix
- 4 — The Power Method, Solution of the Eigenvalue Problem Matrix
- 5 — Finite Differences, Solution of ODE/PDE (BVP) Differential
- 6 — Quadrature, Solution of IVP / Integral Equation Differential/Integral

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(I) Acknowledgements and Recommended Books / Resources

This set of notes is a new and continuing development, for summarizing main ideas and technical formulae (not for substituting lecture attendance), based on the main textbooks for the course

..... HCL reference books for M266

- R. L. Burden & J. D. Faires, Numerical Analysis, PWS-Kent. 1981, 1989, or 1993.
QA297.B94.5 - 4 copies
- K. Atkinson, Introduction to numerical analysis, John Wiley. 1978.
QA297.A87 - 1 copy
- A. Jennings, Matrix Computation, John Wiley, 1977 (1st ed) and 1992 (2nd ed).
QA166.J55.2 - 2 copies

and also following various informal / incomplete notes from several colleagues including

- Early Draft of *Numerical Mathematics and Scientific Computation*, by Germund Dahlquist and Ake Bjorck
- *Numerical Analysis Slides* by Dr Aaron Naiman who used the book *Numerical Mathematics and Computing* by Cheney and Kincaid.
- Sample lecture notes by Dr Brian Doman (the previous M266 lecturer).

Finally I encourage all to

- Search the internet e.g. search **Euler method** from <http://www.google.com>

(II) Course Website / Contact

The website for the course is<http://www.liv.ac.uk/~cmchenke/Math266>

It will contain some notes, homework problems and solutions which you should be able to print out if you miss them in class (these will only appear after a lecture — not before).

The lecturer can be contacted by email: k.chen@liverpool.ac.uk

(III) Homework and Assessment:

Homework will be set fortnightly on Tuesdays and collected in for marking at the start of the first lecture on the following Tuesday, when full solutions will be distributed. Late work will not be accepted. *The homework counts 10% towards the final assessment.* However, the first homework will just be for revision and will not count. Therefore each of the remaining five homeworks will count 4% towards the final mark, and the exam will count for 90%.

(IV) Sick Notes: If you fail to complete a homework on time due to illness you must hand in a sick note to the Departmental Office. Otherwise your mark for the homework which you miss will be counted as zero. There is no September resit available for the homework element of the mark—in other words, in the event of your failing the module, you will be able to resit the exam the following September, but your original homework marks will be carried forward.

(V) TutorialsThe tutorial hour is set at **1pm**, every Tuesday in Room **029**. Any problems with doing the home work should be discussed there.

IMPORTANT NOTE

- Many equations cannot be solved exactly for the simple reason that the solution is not a simple and easily recognizable function.
- In Computational Mathematics, using computers (ie finite precision arithmetic), we can construct a reliable approximation to the true solution.
- Such proposed methods are called **Numerical Methods** — Relatively easy
- Doing a mathematical analysis of numerical methods to establish the reliability leads to the subject of **Numerical Analysis** — Relatively hard
- This Math266 is the only course available in Liverpool that introduces **Numerical Methods** and **Numerical Analysis**. (More NM and Less NA)

IN A NUTSHELL

- Function $f(x), x \in [a, b]$ \longrightarrow Vector $\mathbf{f} \in R^n$
- Differential/Integral $(\mathcal{L}f)(x), x \in [a, b]$ \longrightarrow Matrix $A \in R^{n \times n}$

Purpose of NM / NA :

Problem solved properly as $n \rightarrow \infty$