UNIVERSITY OF LIVERPOOL Management School

Working Paper in Economics

202414

Intangible Cycles

Shalini Mitra*, Gareth Liu-Evans†

*University of Liverpool Management School, Liverpool, L69 7ZH, email:shalini.mitra@liv.ac.uk
†University of Liverpool.

Intangible Cycles

Shalini Mitra[*]
Gareth Liu-Evans[‡]

Current version: September 17, 2024
First version: January 8, 2021

Abstract

This paper investigates the role of an intangible investment technology shock in driving and propagating business cycles. In a dynamic general equilibrium framework with borrowing constrained entrepreneurs, we show that consumption smoothing by entrepreneurs, which is associated with reallocation of physical investment and hours from final goods to intangible investment, is the key mechanism through which aggregate co-movement arises in the model. The reallocation channel is especially strong in the presence of binding financial constraints. We use firm level intangible capital estimates to discipline the model and show that the entrepreneur's degree of risk aversion, which determines their preference for consumption smoothing given their constant relative risk aversion (CRRA) utility, plays a key role in quantitatively generating the observed joint aggregate business cycle dynamics of output, consumption, investment and hours. For instance, entrepreneurs can display too little or too much risk aversion, in which case aggregate comovement is negated.

Keywords: Intangible investment shock, reallocation, intangible capital, business cycles, aggregate comovement
JEL Classifications: E13, E22, E32, O33

[*]University of Liverpool Management School, Liverpool, L69 7ZH, email:shalini.mitra@liv.ac.uk
[†]University of Liverpool
[‡]University of Liverpool

1 Introduction

Firms today devote a larger share of their resources to investment in intangibles in its various forms than did their predecessors. For example, increasing reliance on knowledge capital for profitability and revenue generation has meant more resources dedicated to building intellectual property such as patents, copyrights, and trademarks; similarly, organizational capital requires investments in organizational structures such as management expertise, corporate culture, and information systems; social capital requires investment in building and maintaining relationships with suppliers, customers, and the local community; and human capital requires investments in individual capabilities such as education and training, skills and experience. Figure 1 shows the evolution of estimates of average firm level intangible capital (IC) intensity, which is the ratio of intangible to total (intangible and physical) capital stock of a firm, for Compustat US firms.

There has clearly been an increase in the average firm-level IC intensity since the mid-1980's. However, Figure 1 also shows that this increase is driven by the largest US firms. From the figure, the IC intensity of the largest firms (by their sales-to-GDP ratio) is not only higher than the rest of the firms in the economy, but has been diverging away from the rest since the early 2000s. It is a well established fact in the literature that the largest firms, both in the US and other advanced economies, account for a substantial share of aggregate fluctuations (Gabaix (2011); Carvalho and Grassi (2019); Bijnens and Konings (2020)).

Against this background of the rising importance of intangible capital in production in general, and particularly in the case of large firms, we investigate the role played by an intangible investment technology shock in driving and propagating business cycles. We particularly want to understand the main channels through which such a shock can generate correlated movements in output, consumption, investment and hours via the firm. We do this in a dynamic general equilibrium framework with borrowing constrained, risk averse entrepreneurs optimally allocating physical capital investment across final goods and intangible investment every period. These are the only two sectors of production in the economy, and the sectors are both internal to the firm[1]. The only source of uncertainty in the model is a shock to the technology to produce intangible capital investments within the firm.

[1]Our focus in this paper is on a representative firm as we want to study the main channels of propagation that arise via the firm but the model's results go through in a set-up with two firm types - more and less intangible intensive, with the more intangible intensive type representing the largest firms.

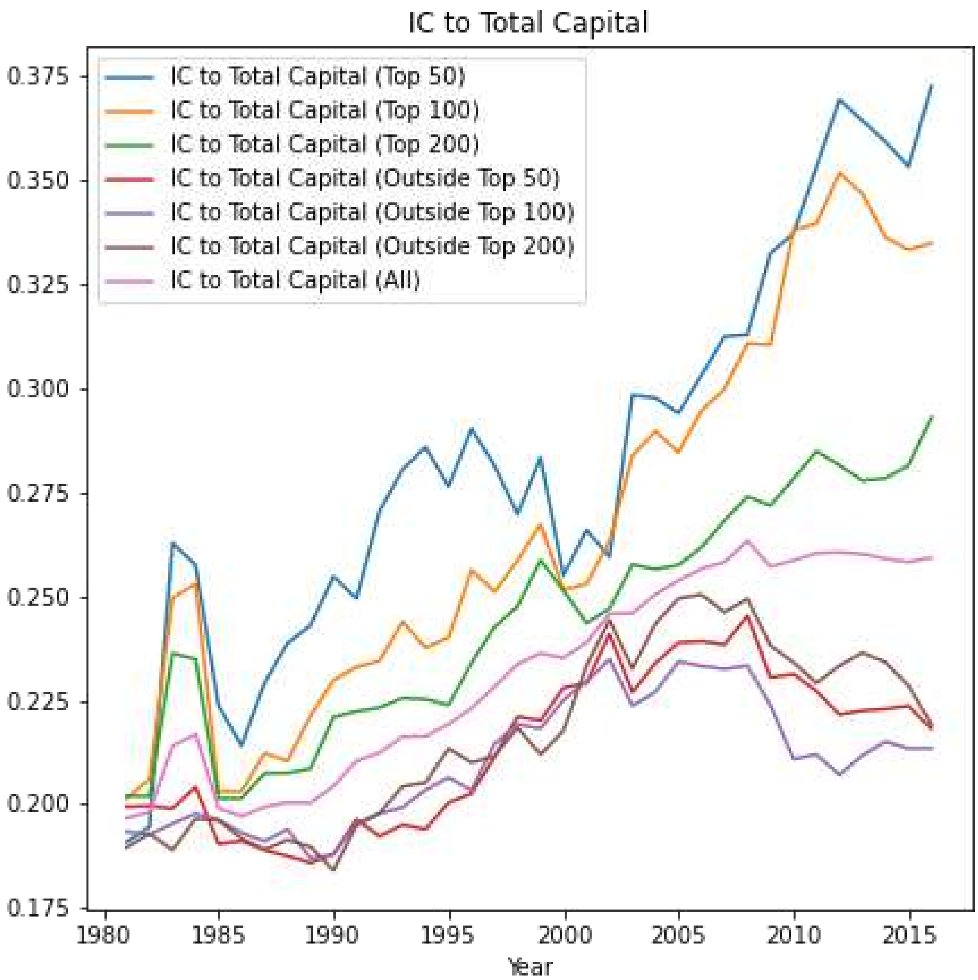

Figure 1: IC intensity is defined as IC to Total Capital (tangible +intangible). Firms are divided into "Top" and "outside Top" according to their sales-to-GDP ratio. Data is from Ewens et al. (2019). The authors use the sum of R&D expenditures and organizational capital spending by US firms in Compustat to arrive at their IC estimates.

Intangible capital in our model, in addition to physical capital and labor, is an input into the production of both final goods and intangible investment (McGrattan and Prescott (2014) and Mitra (2019), among others). A technology shock to intangible investment raises the marginal productivity of factors used in intangible investment relative to final goods. We show that as long as entrepreneurial risk aversion is neither too high nor too low, there is a reallocation of hours and physical investment from final goods to intangible investment. This implies aggregate investment and aggregate hours do not change much upon the shock's impact. These aggregates only rise over time as the stock of IC rises, in the aftermath of the shock. Specifically, as the IC stock increases, hours worked in, and therefore output of, final goods also rise. The increase in final goods hours pushes up aggregate hours worked, while the increase in aggregate (final goods) output, through the aggregate resource constraint, is met with an increase in both aggregate physical investment and aggregate consumption. Aggregate consumption rises as both household consumption and entrepreneurial profits rise. Therefore, aggregate output, aggregate hours, aggregate investment and aggregate consumption all rise jointly in a hump-shaped manner in response to an intangible investment technology shock, generating business cycles.

The degree of risk aversion of the entrepreneurs plays a key role in the model's results. It is the key parameter responsible for generating a temporary reallocation of hours and physical investment from final goods to intangibles as the shock hits. We show that without a reallocation of spending between sectors upon the shock's impact, aggregate comovement doesn't arise. Before we expound on this mechanism it is important to note that we define reallocation as the transfer of resources, in this case, physical investment and hours, between firms or productive technologies. Where no transfer of resources have occurred, even if there is an increase in resources within a particular sector relative to the other sector, there is no reallocation. For example, if physical investment rises in both sectors, but it rises more in intangible investment, there is a relative increase in investment in intangibles, but no reallocation, as investment in final goods does not fall for investment in intangibles to rise, in other words, there has been no transfer of investment spending between sectors. Similarly, if physical investment does not rise in intangibles, but falls in final goods, there is once again, a relative increase in physical investment in intangibles, but no reallocation of resources have occurred.

Since entrepreneurs allocate physical investment optimally across the two sectors every period, a positive shock to the intangible investment producing technology always generates a relative increase in physical investment spending in intangibles. However, a *reallocation* of physical investment spending from final goods to intangible investment only arises when the degree of risk aversion of the entrepreneurs, ι_e, in the entrepreneurs' constant relative risk aversion (CRRA) utility function, lies within a specific range. Without reallocation there is an instantaneous jump in aggregate hours or aggregate investment, or both, as the shock hits, which negates or reverses aggregate comovement of these variables particularly with aggregate consumption.

The key role of ι_e stems from the property that under CRRA utility, it measures the inverse of the intertemporal elasticity of substitution of the entrepreneurs. In other words, ι_e captures the strength of the entrepreneurs' preference for consumption smoothing. A smaller (higher) degree of risk aversion, or, a lower (higher) value of ι_e, implies a greater (lower) tolerance for intertemporal substitution in consumption, such that the preference for consumption smoothing by entrepreneurs is lower (higher), generating a lower (higher) incentive for within-firm reallocation of physical investment spending.

Consumption smoothing by entrepreneurs thus generates a preference for spending reallocation across sectors within the firm. This incentive is especially strong in the presence of binding financial constraints. That is when limited external funding is available to smooth consumption, the internal or within-firm reallocation motive for risk averse entrepreneurs is stronger. It is possible, however, for entrepreneurs to be 'too much' or 'too little' risk averse, generating too little or too much consumption smoothing. For instance, we show that the nested cases of risk neutrality ($\iota_e = 0$) and log utility ($\iota_e = 1$) both generate anomalous correlations, particularly between consumption and investment, but also between consumption and hours.

Risk neutral entrepreneurs do not mind large intertemporal substitutions in consumption (or profits). In this case the relative increase in physical investment in intangibles is accompanied by an increase (or instant jump) in aggregate investment spending as physical investment in both sectors rise. Since aggregate output does not increase upon impact of the shock, the increase in aggregate investment is financed by drawing down consumption spending of the entrepreneurs, which is large enough that aggregate consumption falls, generating a negative aggregate consumption-investment correlation in this case. This phenomenon is akin to the literature on physical investment specific technology (or IST) shocks which typically generate negative aggregate consumption-investment correlations (see Justiniano et al. (2010) for example).

When risk aversion is too high on the other hand, as in the case of log utility (or $\iota_e = 1$), there is too much consumption smoothing by entrepreneurs, so much so that the rise in household consumption, due to the increase in real wage (as marginal productivity of hours jump), causes aggregate consumption to rise upon the shock's impact. Aggregate investment and aggregate hours both fall, given the strong incentive of entrepreneurs to suppress an increase in spending, (wage bill and physical investment), which may negatively affect their profits, causing correlations between both consumption and investment and consumption and hours to turn negative in this case.

Only when entrepreneurial risk aversion is neither too high nor too low, their preference for consumption smoothing generates a reallocation of hours and physical investment from final goods to intangible investment which in turn gives rise to aggregate comovement as highlighted earlier. We quantitatively map out, for various degrees of entrepreneurial risk aversion, the volatilities of, and aggregate comovement among, these key macroeconomic aggregates and find that business cycles are generated when ι_e lies within the plausible range of $0 < \iota_e < 0.75$. In other words, the model quantitatively generates the observed joint business cycle dynamics of output, consumption, hours and investment in response to an intangible investment technology shock for these values of ι_e.

We discipline the model using both aggregate and firm level data on intangible capital and other macroeconomic series for the USA. We particularly use the firm level estimates of Ewens et al. (2019) as in Figure 1, to target the intangible capital intensity of Compustat firms between 1975-2011. We postulate that reallocation of physical investment spending is the main channel through which the joint aggregate business cycle dynamics of the key macroeconomic aggregates arise in response to an intangible investment technology shock. Kehrig and Vincent (2019) find evidence of this channel in the data. They argue that reallocation of investment spending is one of the main ways a firm mitigates the effects of frictions or constraints, whether internal or external, and show that the majority (68%) of firm level investment dispersion in a typical industry occurs within rather than across firms (see also Bachmann and Bayer (2014))[2].

2 Literature and contribution

This paper contributes to two strands of literature - one investigating the sources of business cycle fluctuations and the other studying the rise in importance and implications of intangible capital. The literature on intangible capital so far has focused mainly on the longer term impact of the rise in intangibles - from the fall in labor's income share emphasized in Koh et al. (2020) (see also van Vlokhoven (2024)), to the rise in market concentration of firms in Crouzet and Eberly (2021), to the slowdown in aggregate productivity studied by De Ridder (2024). Very few papers, however, have emphasized the short to medium run effects of intangibles. Some papers that have looked at the implications of including intangible capital in standard macroeconomic models at business cycle frequencies include McGrattan and Prescott (2010), Prescott and McGrattan (2012), Gourio and Rudanko (2014), Mitra (2019). These papers however, do not focus on an intangible investment technology shock as a main driver of business cycles.

The problem usually faced by single non-technology shocks in generating business cycles, is a failure to generate comovement among *all* key macroeconomic aggregates. In their seminal paper, Justiniano et al. (2010) for example, find that physical investment shocks are the main drivers of business cycles, except consumption. Christiano et al. (2014)

[2]While the intangible investment producing sector is internal to the firms in our model by construction, the model's results also go through in a set-up with two firm-types (high and low intangible intensive) - with both within- and between-firm channels of physical investment reallocation.

find that the influence of the investment shock disappears in the presence of a financial accelerator. They find that risk shocks or shocks to the dispersion of firms' productivity instead play the main role. However, their model also does not generate or explain the comovement of aggregate consumption. Becard and Gauthier (2022), building on Christiano et al. (2014), show that by allowing banks to lend to households, in addition to businesses, and simultaneously adjusting lending requirements on the two types of loans, a collateral shock can generate aggregate comovement among output, investment, hours *and* consumption.

In fact, Angeletos et al. (2020) find support for theories of a *single* main business cycle shock, rather than a combination of shocks, that generates strong positive comovement among the key aggregates of output, hours, consumption and investment at business-cycle frequencies. Their empirical findings support "parsimonious theories featuring a main, unifying, propagation mechanism." However, they rule out technology shocks in addition to other commonly used shocks in the literature, such as shocks to investment, risk, uncertainty, news and financing, as candidate drivers of business cycles. This is based on their finding that the main business cycle shock is disconnected from aggregate total factor productivity (TFP) at all frequencies. They emphasize the role of non-inflationary demand shocks instead.

Our model provides a parsimonious theory featuring a main, unifying propagation mechanism, as envisaged by these authors *and* our shock is disconnected to aggregate TFP. However, the intangible investment technology shock at the heart of our model is a sectoral technology shock, which the above authors rule out as a candidate business cycle driver. We are however, not the first to demonstrate how a sectoral technology shock can in fact appear to be disconnected from aggregate TFP. Chahrour et al. (2021) show that sector specific technology shocks can create the appearance of a shock that is orthogonal to aggregate total factor productivity (TFP), even in the absence of any non-TFP shocks. In their model, news that is unrepresentative of the whole economy (even if accurate) generates aggregate fluctuations that are orthogonal to aggregate TFP fluctuations, as firms over- or under-react to actual changes in output.

In our case, it is the largely unmeasured nature of intangible capital, that causes a technology shock to intangible investment to be uncorrelated to aggregate *measured* TFP. Specifically, in our model intangible investments are internally generated within firms and are not measured as investment but rather classed as an expense. Peters and Taylor (2017) (also Ewens et al. (2019)) establish these facts for the universe of Compustat firms in the US. They report that the mean (median) firm in their data purchases only 19% (3%) of its intangible capital externally, meaning the vast majority of a firms' intangible assets are unmeasured, or missing from their balance sheets. Thus, in our model, an increase in intangible investment that occurs due to a technology shock to the intangible investment sector is not registered as an increase in aggregate output and hence not accounted for in measured aggregate TFP. Measured TFP only rises, following the rise in aggregate measured (final goods) output, in a hump-shaped manner, in the aftermath of the shock. Thus our model generates a near zero correlation of measured aggregate TFP with an intangible investment technology shock, which is the single main driver of business cycles in our model environment.

Finally, our paper contributes to the literature on the role of large firms in driving and propagating business cycles. Since the seminal paper of Gabaix (2011), a sizeable literature has established the role of large firms in driving business cycle fluctuations (cite Carvalho and Grassi 2019). From Figure 1 it is clear that the largest US firms by sales-to-GDP ratio, operate with higher average intangible intensities. Furthermore, these firms have recently seen steep increases in both their intangible intensities and sales-to-GDP ratios relative to the rest of the firms in the economy, over the same period. Bajgar et al (2021) shows for instance, that intangibles disproportionately benefit large firms and enable them to scale up and increase market shares.

If intangible capital is more important for large firms and large firms are more important as drivers of business cycles, then clearly an intangible investment shock, by mattering more for these more intangible intensive large firms would have an impact on aggregate business cycle fluctuations. Note that we do not postulate that an intangible investment shock is specific to large firms; rather by being more intangible intensive, the channels outlined in this paper may be more pronounced in these firm-types, and through them, affect aggregate business cycles.

The rest of the paper proceeds as follows. Section 3 sets out the model, Section 4 discusses the calibration strategy, in Section 5 we present the quantitative results of the benchmark model with risk averse entrepreneurs facing binding financial constraints, followed by the case when financial constraints are non-binding and finally, the model version with households as firm owners. We highlight the main differences in mechanisms and results in each of these cases. Section 6 concludes.

3 Model

The economy is composed of firms, owned by borrowing constrained entrepreneurs, and households who lend to entrepreneurs and supply labor to the firms. Firms combine labor, physical and intangible capital to produce two types of goods: final goods and intangible investment. Thus in this framework, firms accumulate and invest in two types of capital - intangible and tangible (or physical). The framework is similar to McGrattan and Prescott (2010), Prescott and McGrattan (2012), Gourio and Rudanko (2014) and Mitra (2019) with the exception that borrowing constrained entrepreneurs and not households are the owners of firms and physical capital. We show that this is an important distinction and present the case of household ownership in Section 5.3.

The **entrepreneurs'** utility is of CRRA type. Every period they maximize profits, which is their deferred consumption (c_e),

$$\text{Max} \sum_{t=0}^{\infty} \upsilon \beta_e^t \frac{c_{e,t}^{1-\iota_e}}{1-\iota_e}. \tag{1}$$

β_e is the entrepreneurs' discount factor and ι_e is the risk aversion parameter such that $\frac{1}{\iota_e}$ gives the intertemporal elasticity of substitution of the entrepreneurs.

The entrepreneurs' profits, which are their deferred consumption is given by,

$$c_{e,t} = y_t - w_t l_t - x_{k,t} - R_t b_{t-1} + b_t. \tag{2}$$

They spend on consumption, wage bill, physical capital investment and loan repayments every period and finance their spending with final goods output and new loans.

The production function is given by:

$$y_t = A_y \left(k_{y,t}^{\alpha} z_t^{\gamma} l_{y,t}^{1-\alpha-\gamma} \right)^{\mu}, \tag{3}$$

where A_y is a parameter measuring the productivity of final goods. k_y, l_y and z are firm level inputs of capital, labor and intangible capital respectively, while $\alpha\mu$ and $\gamma\mu$ are the firm level elasticities of tangible and intangible capital input in production. Note that unlike physical capital and labor, the intangible capital stock, z is not sector-specific within the firm. This feature of intangible capital is well documented in the literature and commonly described as the scaleability of intangible capital (McGrattan and Prescott (2010); Haskel and Westlake (2018); Crouzet et al. (2022)). It captures the fact that intangible capital, such as knowledge, can be used to produce both final goods, such as automobiles, and ideas that improve or increase our knowledge of future final goods production.

Intangible capital accumulates according to,

$$z_{t+1} = (1-\delta_z)z_t + x_{z,t}, \tag{4}$$

where δ_z is the intangible capital depreciation rate and x_z is new investment in intangibles.

x_z also requires labor, tangible and intangible capital for production and has the following production function:

$$x_{z,t} = A_z T_t \left(k_{z,t}^{\alpha} z_t^{\gamma} l_{z,t}^{1-\alpha-\gamma}\right), \tag{5}$$

where A_z is a parameter measuring the productivity of the intangible capital investment producing technology and T_t is an intangible investment productivity shock, which follows an $AR(1)$ process given by,

$$\log T_t = \rho \log T_{t-1} + \epsilon_t, \tag{6}$$

with $\epsilon_t \sim [0, \sigma^2]$.

Tangible capital accumulates according to,

$$k_{t+1} = (1-\delta_k)k_t + x_{k,t} - s\left(\frac{x_{k,t}}{k_t}\right)k_t. \tag{7}$$

δ_k is the depreciation rate of physical capital and $s(.)$ is a capital adjustment cost function defined as in Hayashi (1985):

$$s\left(\frac{x_{k,t}}{k_t}\right) = \frac{\phi}{2}\left(\frac{x_{k,t}}{k_t} - \delta_k\right)^2. \tag{8}$$

The main role of equation 8 is to ensure the benchmark model does not generate too much physical investment volatility relative to the data.

Finally, the entrepreneurs face borrowing constraints as in Buera and Moll (2015),

$$b_t \leq \theta k_t. \tag{9}$$

which implies, they can borrow at most a fraction θ of their physical capital stock in any given period t, due to limited commitment and other underlying frictions.

Households supply labour and lend to entrepreneurs every period. They solve the following problem,

$$\underset{c_{h,t},l_t,b_t}{\text{Max}} \quad E_t \sum_t \beta^t \left(\frac{c_{h,t}^{1-\iota_h}}{1-\iota_h} - \frac{\psi l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right), \tag{10}$$

subject to,

$$c_{h,t} + b_t = w_t l_t + R_t b_{t-1}, \tag{11}$$

where β is the household discount factor, c_h is household consumption and l is total hours worked. ι_h denotes the household's degree of risk aversion, which is fixed at 1 throughout our analysis in this paper, implying the households exhibit log utility. η is the elasticity of labor supply. Equation 11 is the households' budget constraint; they equate their wage income $w_t l_t$ and interest payments on last period's loans $R_t b_{t-1}$, on the right, to consumption $c_{h,t}$ and new loans b_t on the left, every period. We assume, $\beta_e < \beta_h$ such that households are net savers in equilibrium, while the entrepreneurs are net borrowers as we show in Section 3.3.

3.1 Optimality conditions

The firms' first order conditions with respect to physical capital, in the two sectors of final goods and intangible investment respectively are:

$$E_t\Big[\beta_e u'(c_{e,t+1}) \left(1 - \delta_k - s\left(\frac{x_{ky,t+1}}{k_{y,t+1}}\right) + s'\left(\frac{x_{ky,t+1}}{k_{y,t+1}}\right)\frac{x_{ky,t+1}}{k_{y,t+1}} + \alpha\mu\frac{y_{t+1}}{k_{y,t+1}}\right) + \lambda_{t+1}\theta\Big] = u'(c_{e,t})\left(1 + s'\left(\frac{x_{ky,t}}{k_{y,t}}\right)\right), \tag{12}$$

and,

$$E_t\Big[\beta_e u'(c_{e,t+1})\left(1-\delta_k - s\left(\frac{x_{kz,t+1}}{k_{z,t+1}}\right) + s'\left(\frac{x_{kz,t+1}}{k_{z,t+1}}\right)\frac{x_{kz,t+1}}{k_{z,t+1}}\right) + \zeta_{t+1}\alpha\frac{x_{z,t+1}}{k_{z,t+1}} + \lambda_{t+1}\theta\Big] = u'(c_{e,t})\left(1+s'\left(\frac{x_{kz,t}}{k_{z,t}}\right)\right) \tag{13}$$

where λ_t and ζ_t are the Lagrange multipliers associated respectively with the entrepreneur's borrowing constraint in equation 9 and their intangible capital accumulation equation 4.

ζ_t can be thought of as the "shadow" value, to the entrepreneur, of the firm's intangible capital constraint. Equations 12 and 13 equate the marginal cost of acquiring an additional unit of physical capital on the left to its marginal benefit on the right in each sector of production. Note the marginal cost is the same in both equations - a unit of current consumption which is given up plus the associated adjustment cost of the added unit of investment, $s'(.)$. Also same in both equations is the financial constraint term, $\lambda_{t+1}\theta$, on the left, denoting that an additional unit of capital accumulated, irrespective of the sector it is accumulated in, helps relax the financial constraint of the entrepreneur going forward.

However, to the left of equation 12, the marginal benefit of an additional unit of physical capital investment in final goods or $k_{y,t+1}$, is composed of the discounted marginal product of physical capital plus the value to the firm of undepreciated future capital, and the contribution of the new unit of capital to the marginal decline in installation costs in the future. Whereas, the marginal benefit of an additional unit of physical capital investment in the intangible investment sector, or $k_{z,t+1}$, on the left of equation 13 is composed of the marginal product of physical capital weighted by ζ_{t+1}. That is, the contribution to marginal revenue generated from an additional unit of investment in physical capital in the intangible investment sector, depends on the expected value to the firm of its *future* intangible capital constraint.

This highlights the key trade-off faced by an entrepreneur in allocating a unit of physical capital investment across the two sectors of the firm - final goods and intangible investment. The marginal product of new physical capital investment in final goods in period t, is realized the period after, in $t+1$, in the form of new final goods for consumption. In contrast, the marginal product of an additional unit of physical capital investment in intangible investment in period t, relaxes the intangible capital constraint of the firm in $t+1$. This allows the firm to produce more final goods along with new intangible investments in $t+2$. Thus the direct effect of a rise in physical capital investment in t is to relax the IC constraint of the firm in $t+1$.

The marginal product of physical investment in the intangible investment sector, is therefore multiplied by the expected value of a change in the firm's IC constraint next period (in utility terms) or ζ_{t+1}. The rest of the left hand side terms in equation 13, have analogous interpretations to the corresponding terms in equation 12 above.

The first order condition with respect to next period's intangible capital is given by,

$$E_t \left[\beta_e u'(c_{e,t+1}) \left(\gamma \mu \frac{y_{t+1}}{z_{t+1}} \right) + \zeta_{t+1} \left(\gamma \frac{x_{z,t+1}}{z_{t+1}} + (1 - \delta_z) \right) \right] = \zeta_t. \tag{14}$$

Equation 14 states that the current period shadow value of the firm's intangible capital constraint, ζ_t, on the right, equals the expected value of the marginal benefit from having an extra unit of z_{t+1}, on the left, which in turn has two components. First is the contribution of the additional unit of intangible capital to an increase in future output of final goods by the amount of its marginal productivity in this sector. This is multiplied by the discounted marginal utility of a change in future consumption. Second is the contribution of the additional intangible capital to an increase in new intangible investment by the amount of its marginal product in the intangibles sector, multiplied by ζ_{t+1} - the value in utility terms to the firm of a change in the intangible capital constraint, along with the un-depreciated amount of intangible capital.

Optimality conditions for labor in final goods,

$$\mu \left(1 - \alpha - \gamma\right) \frac{y_t}{w_t} = l_{y,t}, \tag{15}$$

and intangibles,

$$\zeta_t \left(1 - \alpha - \gamma\right) \frac{x_{z,t}}{u'(c_{e,t}) w_t} = l_{z,t}. \tag{16}$$

The firm equates the marginal cost of employing an additional unit of labor, on the right to its marginal benefit on the left, in equations 15 and 16. The marginal benefit of an additional unit of labor in final goods is simply the marginal product of labor in this sector while its marginal cost is the real wage. The marginal benefit of an additional unit of labor in intangibles, however, internalizes the effect of this unit of labor on the shadow value of the intangible capital constraint to the firm, ζ_t. The entrepreneur sacrifices current profits to hire additional labor in intangibles which does not generate additional final goods during the period, but relaxes the intangible capital constraint, from which the entrepreneur derives value (in utility terms).

The first order conditions with respect to entrepreneurial and household borrowing are:

$$E_t\left[\lambda_{t+1} + R_t\left(\beta_{e,t+1}u'(c_{e,t+1})\right)\right] = u'(c_{e,t}) \tag{17}$$

$$\beta u'(c_{h,t+1})R_t = u'(c_{h,t}), \tag{18}$$

where equation 18 is the standard Euler equation of households. Finally, the labor supply decision of the household is given by,

$$u'(c_{h,t})w_t = \psi l_t^{\frac{1}{\eta}}.$$

3.2 Definition of equilibrium

An equilibrium in this economy is defined as a sequence of wages, $\{w_t\}_{t=0}^{\infty}$, and interest rates $\{R_t\}_{t=0}^{\infty}$ and corresponding labor inputs in the two sectors $\{l_{y,t}, l_{z,t}\}_{t=0}^{\infty}$ and loans $\{b_t\}_{t=0}^{\infty}$ such that (i) firms maximize profits subject to Eqs.(2) –(8) and households maximize utility subject to Eq.(10) taking as given the exogenous and endogenous states $\{T_t\}$, $\{k_{y,t}, k_{z,t}, z_t\}$ and the price sequences $\{w_t\}_{t=0}^{\infty}$ and $\{R_t\}_{t=0}^{\infty}$, and (ii) capital, labor, goods and bonds markets clear:

$$k_{y,t} + k_{z,t} = k_t \tag{19}$$

$$l_{y,t} + l_{z,t} = l_t \tag{20}$$

$$c_t + x_{k,t} = y_t, \tag{21}$$

where $c_t = c_{e,t} + c_{h,t}$ and $x_{k,t} = x_{ky,t} + x_{kz,t}$

3.3 Sectoral physical capital share

From the Euler equation of households (equation 18) at steady state, $R = 1/\beta$. Substituting R into the entrepreneur's first order condition for borrowing in equation 17, gives $\lambda = \beta - \beta_e$. This standard condition for the Lagrange multiplier associated with the borrowing constraint simply implies that the constraint is binding when $\beta \geq \beta_e$ and not when $\beta = \beta_e$. While we assume binding financial constraints in the benchmark model, the results for non-binding financial constraints are presented in Section 5.2.

From equation (4) at steady state, $x_z = \delta_z z$ which, plugged into equation (11) gives ζ, the Lagrange multiplier associated with intangible capital accumulation. Once we have both ζ and λ, they can be substituted into the entrepreneur's first order conditions with respect to physical capital in the two sectors - equations 12 and 13 - to arrive at the sectoral physical capital ratio:

$$\frac{k_z}{k_y} = \frac{\gamma}{1-\gamma}. \tag{22}$$

The ratio depends exclusively on γ - the income share of intangible capital in aggregate output, and is less than 1. In other words, the amount of physical capital investment allocated to intangible investment production relative to final goods, depends on the output elasticity of intangible capital, γ alone. The literature estimates a value for γ around 0.1. For instance, Corrado et al. (2009) find that including intangible assets lowers the labor share of income from about 70% to 60% in the U.S. data, and raises capital's share from about 30% to 40%.

4 Parametrization

We calibrate the model to the US economy at an annual frequency. Table 1 presents the calibrated parameters. The household discount factor, β is set equal to 0.96 implying a quarterly interest rate target of 1%. The depreciation rate of physical capital, δ_k, is set equal to its standard annual value of 0.1. The depreciation rate of intangible capital has been the source of much debate in the literature with the main difficulty being a lack of available estimates and widely varying rates of intangible capital depreciation depending on its type and industry. Corrado et al. (2009) for instance use an annual rate of 33% for computerized information, 15% for R&D, 60% for advertising and 40% percent for firm specific resources. Furthermore, when focusing on organizational capital, Ewens et al.

(2019) find huge cross-sectional variation in its depreciation rate across industries, varying from 19% in consumer industries to 49% in healthcare. Given the literature, we assume a benchmark annual depreciation rate for intangible capital of $\delta_z = 0.4$. In Section 5.4 we present results for lower and higher values of δ_z and show that the model's results are not sensitive to assumed values of δ_z.

Following Corrado et al. (2009), we set the income share of physical capital α to 0.3 and the income share of intangible capital to $\gamma = 0.1$, in order that the total share of capital in income is 40% of aggregate returns with labor garnering the remaining 60%. While intangible investment production has a constant returns to scale technology in the model, final goods exhibit decreasing returns, with the returns to scale parameter, μ, set to a standard value of 0.85.

Both households and entrepreneurs display constant relative risk aversion (CRRA) utility functions with risk aversion parameters ι_h and ι_e respectively. However, we fix the value of ι_h to 1, such that households in the model always display log-utility. ι_e plays a key role in the model's results and its value is is varied in the next section, from 0 to 1.

The Frisch elasticity of labor supply is set to $\eta = 5$ following Aoki and Nikolov (2015) and is within the range used in macroeconomic studies (see for instance Gertler and Kiyotaki (2010)). A lower η does not alter the model's results but generates between 5-10% lower employment volatility. Given η, the dis-utility parameter for labor supply, ψ, is chosen to target a total steady state hours worked of 0.33.

The productivity of the intangible investment sector, A_z, is an important parameter and we use it to target the average firm-level intangible intensity in the data, defined as the share of intangible capital in the total capital ($tangible + intangible$) stock. We set the productivity of the final goods sector A_y as the numeraire without loss of generality. We then use the estimates of Peters and Taylor (2017) for intangible capital intensity of Compustat firms (barring regulated utilities, financial firms and firms categorized as public service, international affairs, or non-operating establishments) between 1975-2011 to calibrate A_z. These authors' intangible capital estimates include balance sheet intangibles - those that are externally acquired (purchased) by firms and non-balance sheet or internally generated intangibles such as knowledge and organization capital. Their sample has a mean intangible intensity of 43% in a typical firm-year. However, an average of 19% of intangibles is acquired externally by their mean firm. Given our focus on internally generated intangibles, we use A_z to target a slightly lower intangible capital intensity (37%) in the benchmark model, than that obtained by the authors. However, higher intangible capital intensities do not change the results of the model.

To allow for binding financial frictions, the discount factor of the entrepreneur is set at a value lower than the household's, such that $\beta_f = 0.90$. We alternatively set $\beta_f = \beta = 0.96$ in Section 5.2 to study the effect of non-binding financial frictions. Given β_f, the financial constraint parameter, θ is chosen to target an average non-financial business-debt-to-income ratio of 0.76 between 1990-2020 from the Financial Accounts of the United States - Z.1 (obtainable at: $https : //www.federalreserve.gov/releases/z1/dataviz/z1/nonfinancial_debt/char$ $growth; series : business$).

Since the intangible investment shock is the only source of disturbance in the model, we set the standard deviation of innovations to the intangible investment shock, σ_t to a value that generates the post-1980's output volatility in US data or an average standard deviation of output of 1.4. Recall this was a period of markedly lower aggregate volatility in the US as documented by a large literature (see for instance, Stock and Watson (2002) and Galí and Gambetti (2009) among others). We assign the persistence parameter, ρ_t a value of 0.85 in keeping with a quarterly persistence rate of 0.96 used for productivity shocks in the literature.

Table 1: Parameter Values

Parameter	Value	Target
Household discount factor, β	0.96	Annual interest rate of 4%
Entrepreneur discount factor, β_f	0.90	Literature
Phys. Capital income share, α	0.30	Corrado et al. (2009)
Int. Cap. income share, γ	0.1	Corrado et al. (2009)
Phys. Capital depreciation rate, δ_k	0.1	Literature
Phys. capital adj. cost param, κ	0.5	Phy. inv. vol. 3.5 times output vol.
Financial constraint parameter, θ	0.5	Non-financial business debt-to-income
IC depreciation rate, δ_z	0.4	Ewens et al. (2019) and Corrado et al. (2009)
Household risk aversion, ι_h	1	Log-utility
Entrepreneur risk aversion, ι_e	0.25	Benchmark (varies in the model)
Frisch elasticity, η	5	Aoki and Nikolov (2015)
Labor disutility, ψ	3.5	Hours worked=0.35
Returns to scale parameter, μ	0.85	Literature
Final goods productivity, A_y	1	Numeraire
IC investment productivity, A_z	3	IC to total capital ratio of 37%
S.D. of shock, σ	0.1	Post-1980's US output volatility of 1.4
Persistence of shock, ρ	0.85	Equivalent to quarterly persistence of 0.96 for tech. shocks

5 Results

We present the results of a one standard deviation shock to the technology for producing intangible investment. Entrepreneurs are borrowing constrained in this section. We allow financial constraints to be non-binding in Section 5.2, and finally, present and discuss the case for households as owners of firms and physical capital in Section 5.3. We adjust parameter values in each case to preserve the macroeconomic and firm-level targets in Section 4 wherever relevant.

5.1 Effect of an intangible investment technology shock

Table 2 presents the main results of the model for varying degrees of entrepreneurial risk aversion ι_e, where $\iota_e \in [0, 0.1, 0.25, 0.5, 0.75, 1]$. Note firstly, that volatility of aggregate output, aggregate physical investment and aggregate hours rise, while aggregate consumption volatility falls as entrepreneurs' risk aversion increases. This is straightforwardly explained by the fact that higher risk aversion is associated with a greater degree of consumption smoothing by the entrepreneurs. Since entrepreneurs smooth consumption by adjusting physical investment and hours internally across final goods and intangible investment, higher values of ι_e are associated with rising physical investment and hours volatility as well as a strengthening correlation of aggregate hours with aggregate investment. These mechanisms are discussed in detail below. Note for now that barring the lowest value of $\iota_e = 0$ and its highest values $\iota_e = [0.75, 1]$, the model successfully generates comovement among the key macroeconomic aggregates of output, investment, consumption and hours in Table 2. The impulse responses in the first two rows of Figure 2 also highlight the hump-shaped nature of these variables' responses to the shock.

Table 2: Business cycle moments

	(1)	(2)	**(3)**	(4)	(5)	(6)
ι_e	0	0.1	**0.25**	0.5	0.75	1
$vol(y)$	1.38	1.39	**1.44**	1.49	1.52	1.54
$vol(x_k)$	4.19	3.17	**5.07**	7.94	9.71	10.9
$vol(c)$	1.85	1.38	**0.93**	0.58	0.49	0.51
$vol(l)$	0.28	0.55	**0.87**	1.18	1.36	1.47
$corr(x_k, y)$	-0.09	0.60	**0.92**	0.95	0.96	0.97
$corr(c, y)$	0.91	0.94	**0.92**	0.67	0.22	-0.16
$corr(l, y)$	0.85	0.95	**0.95**	0.96	0.96	0.96
$corr(c, x_k)$	-0.49	0.3	**0.68**	0.41	-0.06	-0.4
$corr(c, l)$	0.61	0.87	**0.8**	0.44	-0.05	-0.41
$corr(l, x_k)$	0.33	0.63	**0.95**	0.99	1	0.99
$corr(T, TFP)$	-0.2	-0.15	**-0.11**	-0.06	-0.03	-0.01

Model-implied moments in response to a one standard deviation shock to the intangible investment. ι_e is risk aversion of entrepreneurs. All variables are as defined in the model. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

Focusing initially on the benchmark case of $\iota_e = 0.25$ in Figure 2, the main mechanism that drives the correlated hump-shaped response of the key macroeconomic aggregates to the shock is as follows: Marginal productivity of physical capital and labor, in the intangible investment sector, rise relative to final goods, as seen in panels (l)-(o). Physical investment and hours in intangible investment rise as a result (panels (f) and (h)). Given the positive degree of risk aversion at $\iota_e = 0.25$, entrepreneurs dislike large intertemporal fluctuations in consumption, implying, they reallocate physical investment spending from final goods to intangibles in response to the shock rather than increase aggregate investment spending which would need to be financed by drawing down current consumption (since final goods production has not risen yet).

Thus physical investment in final goods falls in panel (g) while physical investment in intangibles rises as seen in panel (f). The same consumption smoothing motive causes an increase in hours in intangible investment in panel (h) to be accompanied by a fall in hours in final goods in panel (i). The large increase in marginal productivity of labor in intangible investment (MPL_z), due to the shock, drives up intangible investment hours, l_z as well as the real wage. Thus the wage bill of the entrepreneurs rise, pushing down their profits. Risk averse entrepreneurs reallocate hours from final goods to intangible investment in order to prevent this from happening in response to the shock. The fall in

l_y however, pushes up the marginal productivity of labor in final goods(MPL_y), due to diminishing returns, causing the initial jump in real wage to be even higher as wages are equalized across sectors.

Thus the more risk averse the entrepreneurs, the more *aggregate* hours has to fall in response to the shock in order to lower the rise in the wage bill and hence, the fall in current profits (and consumption) of entrepreneurs. This can be observed from panels (e), (h) and (i) where aggregate hours fall more the more risk averse the entrepreneurs are, and this is achieved by both final goods hours falling more and hours in intangible investment rising less in response to the shock. The drop in hours worked in final goods causes final goods output, and hence aggregate investment and aggregate consumption to fall as the shock hits.

As the intangible capital stock rises next period however, easing the intangible capital constraint of the firm, final goods hours and output increases, which causes aggregate consumption and aggregate investment to rise in turn, driven by higher profits of the entrepreneurs and rising physical investment in final goods respectively. The rise in final goods hours pushes up aggregate hours in the periods following the shock. Thus the key macroeconomic aggregates of output, consumption, investment and hours rise in a correlated fashion over time, in response to an intangible investment technology shock in our model.

Note that given intangible capital is an input into final goods, there is an incentive for entrepreneurs to raise physical investment in final goods too as the shock hits intangible investment, since final goods output rises, eventually, after the shock. However, as explained in Section 3.1, a unit of physical investment in the intangible investment sector raises final goods output for consumption with a further delay than the same unit invested in final goods directly.

Therefore a more risk averse entrepreneur, with preference for consumption smoothing, when faced with an increase in the marginal productivity of physical capital in intangible investment relative to final goods, has a greater preference for increasing physical investment in final goods at a later period, when the stock of intangible capital has already increased. This allows them to stem a large drop in current consumption or profits by reallocating spending between sectors.

As risk aversion of the entrepreneur declines however, and the motivation to smooth consumption diminishes, they suppress increases in aggregate investment less while tolerating larger drops in their profits, and hence consumption. Thus physical investment rises more in the intangible investment sector while falling less in final goods, lowering physical investment reallocation within the firm. When $\iota_e = 0$ for instance, that is, entrepreneurs are risk neutral, physical investment rises in both sectors upon the shock's impact causing aggregate investment spending to jump.

In this case, entrepreneurial profits fall not only due to the increase in aggregate investment spending but also due to a jump in the wage bill, as aggregate hours do not fall much given the entrepreneurs' higher tolerance for intertemporal fluctuations in consumption (profits). The large drop in entrepreneurial profits in Figure 2 causes aggregate consumption to fall while aggregate investment rises producing a strong negative correlation between consumption and investment at $\iota_e = 0$. From Table 2, this correlation is -0.49.

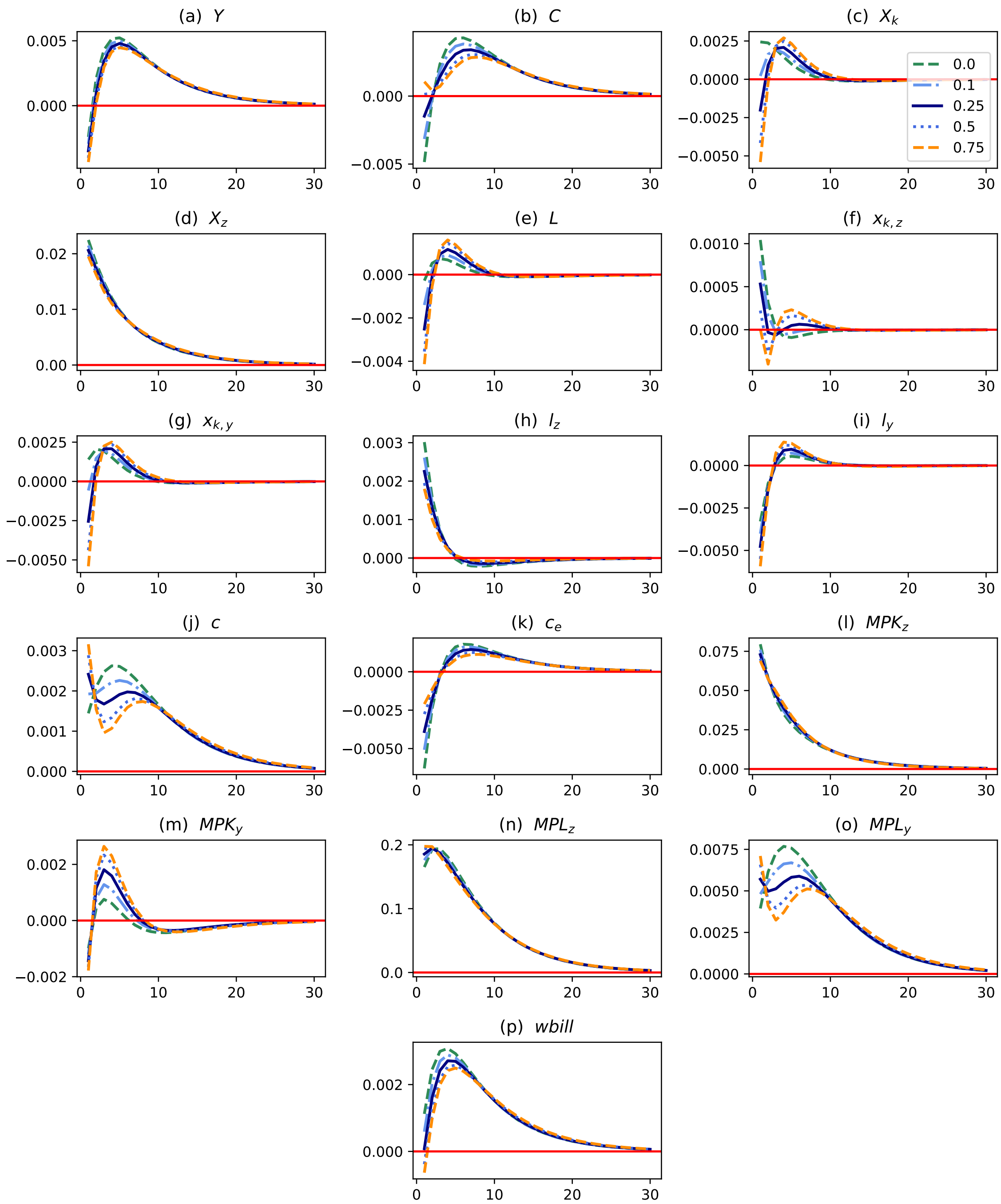

Figure 2: Impulse responses to a one standard deviation shock to intangible investment, for different values of ι_e. We denote aggregate output, aggregate consumption, aggregate hours and aggregate tangible and intangible investment, Y, C, L, X_k and X_z respectively. The rest of the variables are as defined in the paper. Responses are percent deviations from steady state.

As risk aversion of the entrepreneurs rise, reallocation of physical investment and hours within the sector ensures comovement among the macroeconomic aggregates, particularly between aggregate consumption and aggregate investment. This can be seen in Table 2, for $\iota_e = [0.1, 0.25, 0.5]$. Note however that the correlation between the aggregates of consumption and investment as well as that between consumption and hours rise with the value of ι_e, before falling again as ι_e continues to rise. At $\iota_e = 0.75$ for example, both these positive correlations are negated while at $\iota_e = 1$, or the case of log utility for entrepreneurs, these correlations turn strongly negative. We explain the reasons for this below.

At these higher values of entrepreneurial risk aversion, the need to smooth consumption or profits is very strong. This means entrepreneurs strongly dislike any fall in current profits. $l_{z,t}$ still rises upon impact of the shock, but note from Figure 2 that at $\iota_e = 0.75$, $x_{kz,t}$ or physical investment in intangible investment, does not change when the shock hits. This is despite the fact that $x_{ky,t}$ - physical investment in final goods, registers a large drop. Thus physical investment reallocation between sectors is impaired once again, although there is still a relative increase in physical investment in intangibles.

From equations 13 and 16 it is clear that while marginal productivity of $l_{z,t}$ rises immediately upon impact of the shock, marginal productivity of physical capital investment in intangibles, $k_{z,t+1}$, is only realized next period. Thus hours worked in intangibles rises, which as explained earlier causes the wage rate and the wage bill to rise, necessitating a drop in final goods hours. The fall in final goods hours is stronger the greater the risk aversion of entrepreneurs and therefore the larger is the drop in final goods output upon the shock's impact. A larger drop in final goods hours and hence output is manifested as a larger decrease in physical investment spending which is driven mainly by a larger drop in physical investment in final goods.

Thus the fall is larger in aggregate investment and smaller in aggregate consumption, the higher the risk aversion of entrepreneurs, as seen in panels (b) and (c) of Figure 2. In fact at $iota_e = 0.75$ and $iota_e = 1$, aggregate consumption registers increases, since entrepreneurs adjust their consumption little (panel k) upon impact of the shock at these higher levels of risk aversion, while household consumption registers a larger jump (panel j) due to the large increase in the wage rate. More specifically, from panel (o), the larger the entrepreneurial risk aversion, the greater is the increase in MPL_y upon impact, given diminishing returns, which in turn causes the real wage to rise more, driving up household consumption more upon impact.

At $\iota_e = 0.75$, the increase in household consumption is already larger than the decrease in entrepreneurial profits, causing aggregate consumption to generate an increase upon the shock's impact. This increase in aggregate consumption is even larger upon impact for $\iota_e = 1$ which is not shown in Figure 2. Since aggregate investment and aggregate hours both fall, in contrast to aggregate consumption, a negative correlation arises between aggregate consumption and the aggregates of investment and hours resepectively. From

Table 2, at $\iota_e = 0.75$ the correlation between consumption and investment is -0.06 while the correlation between consumption and hours is -0.05. These correlations become even stronger at -0.40 and -0.41 respectively, when $\iota_e = 1$.

5.1.1 Total factor productivity and the intangible investment shock

From Table 2 the correlation between aggregate TFP and the intangible investment technology shock, T, ranges from -0.20 for $\iota_e = 0$, to -0.03 for $\iota_e = 0.75$, that is, the correlation goes from mildly negative to zero. This disconnect between aggregate TFP with what is essentially a TFP shock to intangible investments, is driven by our assumption that intangible investments are unmeasured. In other words, as highlighted in Peters and Taylor (2017), intangible investments, which are generally internally produced by firms, are expensed rather than measured as investment spending in firm balance sheets. This is reflected in equation (2) of our model, where the firm's production of intangible investments does not appear in the calculation of its profits.

Thus, while the shock causes an immediate jump in intangible investment production, this increase is not recorded as an increase in output. Final goods, or measured output, only rises over time in response to the shock as the stock of intangible capital rises, as explained in the previous section. Measured TFP, which follows the same trajectory as final goods (see Figure 4 in Section 5.4) in response to the shock, thus has an almost zero correlation with the intangible investment technology shock in our model. We show in Section 5.4, that this correlation actually varies from negative to mildly positive depending on the assumed depreciation rate of intangibles, although other model results remain unaltered.

Thus a shock to IC investments as the single source of disturbance in our model is capable of generating business cycles, while still being disconnected from measured aggregate TFP. This last has been evidenced by Angeletos et al. (2020) as a key feature of any single main driver of business cycles and forms the basis of the authors' ruling out TFP shocks (as well as other well known shock such as investment, news and financial shocks) as possible business cycle drivers. As in Chahrour et al. (2021) (discussed earlier), however, we offer another instance of a case when sector specific technology shocks can create the appearance of a shock that is orthogonal to aggregate *measured* TFP, even when non-TFP shocks are absent.

5.2 Role of financial frictions

Table 3 presents results for the case when financial frictions are not binding. The dynamics of the model are largely similar whether or not financial frictions are binding. That is, depending on the entrepreneur's risk aversion, the intangible investment shock causes reallocation of investment spending and hours from final goods to intangible investment which produces aggregate comovement among output, hours, consumption and investment as before.

Table 3: Business cycle moments: non-binding financial constraints

	(1)	(2)	**(3)**	(4)	(5)
ι_e	0	0.1	**0.25**	0.5	0.75
$vol(y)$	1.4	1.43	**1.48**	1.52	1.54
$vol(x_k)$	4.07	3.19	**5.27**	7.48	8.63
$vol(c)$	2.03	1.28	**0.75**	0.48	0.49
$vol(l)$	0.32	0.64	**0.96**	1.21	1.33
$corr(x_k, y)$	-0.06	0.76	**0.95**	0.97	0.97
$corr(c, y)$	0.88	0.93	**0.92**	0.36	-0.15
$corr(l, y)$	0.79	0.95	**0.96**	0.96	0.97
$corr(c, x_k)$	-0.52	0.46	**0.65**	0.12	-0.38
$corr(c, l)$	0.44	0.83	**0.7**	0.1	-0.04
$corr(l, x_k)$	0.49	0.82	**0.98**	0.99	1
$corr(T, TFP)$	-0.2	-0.16	**-0.12**	-0.08	-0.05

Model-implied moments in response to a one standard deviation shock to the intangible investment production technology. ι_e is risk aversion of entrepreneurs. All variables are as defined in the model. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

In the absence of binding financial constraints however, risk averse entrepreneurs, can smooth consumption more effectively. Thus the correlation between aggregate consumption and investment starts to fall at a lower level of entrepreneurial risk aversion when financial frictions are not binding compared to when the are. At $\iota_e = 0.5$, greater consumption smoothing leads to a consumpion-investment correlation of only 0.12 compared to a correlation of 0.41 in when frictions were binding (see Table 2). While $\iota_e = 0.75$ now generates a strong negative correlation of -0.38 between the two aggregates, compared to only -0.06 in the case with binding frictions.

Thus reallocation of investment spending between sectors, with or without binding financial frictions, is key to generating business cycle dynamics in the model. The instances when the reallocation channel is not the main mechanism behind the model's dynamics, that is when the entrepreneur is either too little or too much risk averse, are also the cases when the model fails to generate aggregate comovement. We next look at the version of the model with household ownership of firms and physical capital as the literature on intangible capital so far has mainly focused on this version (McGrattan and Prescott (2010), Prescott and McGrattan (2012), Gourio and Rudanko (2014), Mitra (2019)). These models do not have a reallocation channel for physical investment with physical capital reallocation being the only channel of firm level adjustment in response to the shock. We show that such a model fails to generate business cycles. Specifically, this model does not generate comovement among the key macroeconomic aggregates, in addition to generating too little volatility of consumption and investment.

5.3 Household ownership of firms and physical capital

In this section, households own the firms and rent physical capital to them in addition to supplying labor. Firms simply allocate physical capital optimally between the sectors of final goods and intangible investment.

The firm's problem:

$$\text{Max} \sum_{t=0}^{\infty} M^t d_t. \tag{23}$$

$M_t = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor. The equations below are the firm's profits or dividends, which it pays to the household every period, its production function for final goods and intangible investments, its intangible capital accumulation equation and the intangible investment shock, respectively.

$$d_t = y_t - w_t l_t - (r_t + \delta_k) K_t, \tag{24}$$

$$y_t = A_y \left(k_{y,t}^{\alpha} z_t^{\gamma} l_{y,t}^{1-\alpha-\gamma} \right), \tag{25}$$

$$x_{z,t} = s_t A_z \left(k_{z,t}^{\alpha} z_t^{\gamma} l_{z,t}^{1-\alpha-\gamma} \right), \tag{26}$$

$$z_{t+1} = (1-\delta_z) z_t + x_{z,t}, \tag{27}$$

$$\log T_t = \rho \log T_{t-1} + \epsilon_t, \tag{28}$$

where $\epsilon_t \sim [0, \sigma^2]$.

The first order conditions w.r.t physical capital in final goods and intangibles respectively are:

$$\alpha\mu\frac{y_t}{k_{y,t}} - (r_t + \delta_k) = 0, \quad (29)$$

$$\zeta_t(\alpha\frac{x_{z,t}}{k_{z,t}}) - (r_t + \delta_k) = 0, \quad (30)$$

Here the firm equates the cost of renting capital from the household each period plus the depreciation, to its marginal benefit, that is the marginal product of physical capital in each sector. The marginal product of physical capital in intangible investment is weighted by the current value to the firm of the intangible capital constraint equation 4.

Since the firm only gets to choose the allocation of current period physical capital, rather than next period's, only the value to the firm of increasing physical capital in the intangible investment sector by an additional unit this period is relevant here. This is unlike the entrepreneur's physical investment decision in equation 13, where the expected value of a relaxed 'future' intangible capital constraint mattered for how much physical investment went into each sector. In contrast, only the current value to the firm of the intangible capital constraint matters here.

First order condition with respect to next period's intangible capital and hours in the two sectors are,

$$M_{t+1}\left(\gamma\mu\frac{y_{t+1}}{z_{t+1}}\right) + \zeta_{t+1}\left(\gamma\frac{x_{z,t+1}}{z_{t+1}} + (1-\delta_z)\right) = \zeta_t \quad (31)$$

$$\mu\left(1-\alpha-\gamma\right)\frac{y_t}{w_t} = l_{y,t}, \quad (32)$$

$$\zeta_t\left(1-\alpha-\gamma\right)\frac{x_{z,t}}{w_t} = l_{z,t}. \quad (33)$$

Household's problem:

$$\underset{c_t, l_t}{\text{Max}} \quad E_t \sum_t \beta^t \left(\frac{c_t^{1-\iota_h}}{1-\iota_h} - \frac{\psi l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right), \quad (34)$$

subject to,

$$c_t = w_t l_t + (r_t + \delta_k)k_t + d_t. \tag{35}$$

The household's labor supply decision and Euler equations are respectively,

$$u'(c_t)w_t = \psi l_t^{\frac{1}{\eta}}, \tag{36}$$
$$\beta u'(c_{t+1})(r_{t+1} + 1) = u'(c_t). \tag{37}$$

The market clearing conditions are given by equations (19), (20) and (21) as before.

Households earn wages from labor as before and the rental rate r_t on physical capital.

We calibrate the model to target the same variables as in Section 5.1 where possible. The risk aversion parameter, ι, of households, who are the investors in this case, is allowed to take three values, such that, $\iota = [0.5, 0.75, 1]$. Figure 3 presents the impulse responses.

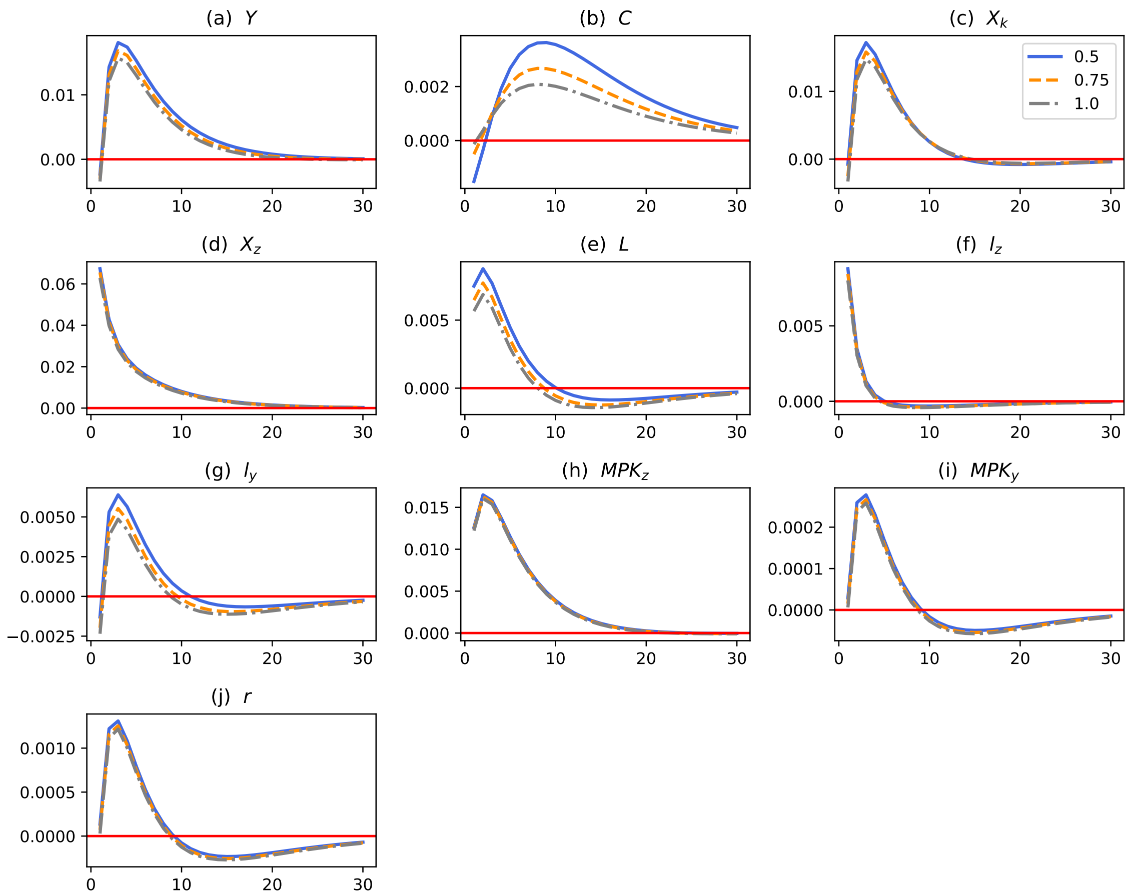

Figure 3: Impulse responses to a one standard deviation shock to intangible investment, for different values of ι, the household/investor risk aversion parameter. We denote aggregate output, aggregate consumption, aggregate hours and aggregate tangible and intangible investment, Y, C, L, X_k and X_z respectively. The rest of the variables are as defined in the paper. Responses are percent deviations from steady state.

Note firstly, in Figure 3, aggregate hours, L_t, jumps immediately upon impact of the shock (in contrast to Figure 2), while aggregate output, aggregate investment and aggregate consumption only rise over time. Furthermore, output and physical investment peak earlier compared to Figure 2. The explanation for both differences with the previous sections where the entrepreneurs owned the firms and physical capital, lies in firms reallocating physical capital rather than physical investment across sectors in this section.

Note that an increase in intangible investment affects the rental rate of physical capital. From equations (29) and (30), the rental rate equalizes the marginal product of capital across the two sectors. Physical capital demand in final goods, however, plays a much larger direct role in the determination of r_t, compared to its demand in the production of intangible investments. Therefore, in Figure 3 the rental rate does not jump, although MPK_z does in response to the shock. r_t reflects the dynamics of $MPK_{y,t}$ more closely - rising over time before returning to steady state although being higher in value than

MPK_y due to the increase in MPK_z.

The jump in hours in intangibles along with the immediate one-time increase in physical capital in intangibles, as firms reallocate physical capital across sectors, implies intangible investment rises within the same period the shock hits. The stock of intangible capital increases the period after, which is when final goods production also rises. While reallocation of physical investment spending to intangibles in the previous sections lengthens the period over which final goods production increases, thereby delaying the peak in its production, the shorter time frame over which final goods production is postponed when firms reallocate physical capital only, prevents hours worked in final goods from dropping much as the shock hits in this case. Thus while hours rise in intangibles in panel (f) (as before), they do not fall much upon impact of the shock in panel (g) (in contrast to Section 5.1). Thus aggregate hours jumps in panel (e), in response to the shock generating a strong negative correlation between aggregate hours, and aggregate consumption in particular, as seen in Table 4.

Thus physical capital reallocation is associated with an earlier increase in final goods output (compared to when physical investment is reallocated in Section 5.1), which necessitates a quicker increase in physical investment in final goods, reflected in the steep increase in aggregate physical investment in panel (c) of Figure 3. Aggregate consumption which, from the aggregate resource constraint is simply aggregate output less aggregate investment therefore rises more smoothly in panel (b). The smoother increase in aggregate consumption coupled with the jump in aggregate hours in this section, upon impact of the shock generates a strong negative consumption-hours correlation.

This is especially the case when there are no capital adjustment costs (CACs). In fact comovement generated by this version of the model improves with strong CAC as shown in Table 4. However, notice that the case for CACs is ill justified in this case given the model without CACs already generates too little volatility of investment (1.45 times that of output) while aggregate consumption volatility varies between 30-60% of aggregate output volatility. Needless to say, consumption and investment volatility worsens once CACs are included. With a CAC parameter $\kappa = 2$, the correlation between aggregate consumption and hours improves especially for the non-zero values of ι, but not by much. The correlation rises from a low of 0.16 at $\iota = 0.5$, to a mild value of 0.27 at $\iota = 0.75$, before going back down to -0.27 at $\iota = 1$.

As expected the CACs slow down the response of aggregate investment, causing aggregate investment to be smoother and aggregate consumption to be less smooth in their response to the shock. Aggregate hours also exhibits more of a hump shape after the initial jump driven by rising hours in intangible investment. These effects culminate in improved aggregate correlations of aggregate hours and consumption in the presence of CACs in this set-up as seen in Table 4.

Table 4: Business cycle moments with households as firm-owners

	no CACs				with CACs			
ι	0	0.5	0.75	1	0	0.5	0.75	1
$vol(y)$	1.74	1.28	1.24	1.22	1.46	1.2	1.28	1.17
$vol(x_k)$	6.9	1.86	1.79	1.76	2.07	1.66	1.66	1.71
$vol(c)$	14.3	0.62	0.38	0.27	3.12	0.4	0.29	0.27
$vol(l)$	1.28	1.19	1.08	1.01	1.1	0.9	0.84	0.96
$corr(x_k, y)$	0.09	0.99	1	1	0.73	1	1	1
$corr(c, y)$	0.27	0.28	0.41	0.5	0.43	0.75	0.74	0.5
$corr(l, y)$	0.76	0.53	0.5	0.49	0.65	0.49	0.47	0.45
$corr(c, x_k)$	-0.93	0.12	0.32	0.44	-0.3	0.69	0.7	0.44
$corr(c, l)$	-0.37	-0.57	-0.41	-0.27	-0.37	0.16	0.27	-0.27
$corr(l, x_k)$	0.69	0.64	0.57	0.53	0.96	0.52	0.48	0.49

Model-implied moments in response to a one standard deviation shock to the intangible investment production technology. ι is the CRRA risk aversion parameter of households/investors. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

5.4 Sensitivity analysis: Depreciation of intangibles

Recall, from Section 4, that intangible capital depreciation rates vary widely from 20-60% depending on the type of intangible capital, and for the same type of intangible, across industries. Brands, for example, have a depreciation rate of 60% according to Corrado et al. (2009), while the depreciation rate of organization capital varies dramatically across industries going from 19% in consumer industries to 49% in healthcare according to Ewens et al. (2019). They also find that knowledge capital (or R&D) goes from a low of 0.30 in “other” industries to a high of 0.46 in high-tech firms, much higher numbers than traditionally used for R&D in the literature. In this section we study the effect of these widely different depreciation rates on the model's results. We particularly allow the depreciation rate to vary from 0.3 to 0.6 in Table 5. The benchmark case of Section 5.1 is highlighted in bold for comparison.

Table 5: Business cycle moments: varying intangible depreciation

δ_z	0.3	0.4	0.5	0.6
$vol(y)$	1.3	**1.44**	1.56	1.67
$vol(x_k)$	3.69	**5.07**	6.29	7.41
$vol(c)$	0.99	**0.93**	0.88	0.84
$vol(l)$	0.76	**0.87**	1.14	1.4
$corr(x_k, y)$	0.8	**0.92**	0.93	0.94
$corr(c, Y)$	0.95	**0.92**	0.88	0.84
$corr(l, y)$	0.95	**0.95**	0.94	0.94
$corr(c, x_k)$	0.7	**0.68**	0.65	0.61
$corr(c, l)$	0.84	**0.8**	0.75	0.69
$corr(l, x_k)$	0.95	**0.95**	0.95	0.95
$corr(T, TFP)$	-0.35	**-0.11**	0.05	0.18

Model-implied moments in response to a one standard deviation shock to the intangible investment production technology. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

The model's results are largely unaffected by the rate of intangible capital depreciation. The only difference is in the correlation of the intangible investment shock (T) with measured TFP in the final row. At a relatively lower annual depreciation rate of 0.3 the correlation is quite negative, falling to near zero and then becoming mildly positive as the depreciation rate rises to the highest values observed in the literature. From Figure 4, it is clear that measured TFP similar to final goods output rises over time before peaking and falling back to steady state, generating an almost zero correlation with the one time positive technology shock that causes an increase in intangible investment (panel b) that is unmeasured in aggregate output and therefore in aggregate TFP.

A higher rate of intangible capital depreciation is associated with a lower incentive to reallocate physical investment from final goods to intangible investment and hence a steeper increase in final goods output upon impact of the shock. Intangible investment rises less with the shock while final goods rise and peak sooner after the shock's impact as seen in Figure 4, for higher rates of depreciation. The smaller initial increase in intangible investment also causes final goods to fall more quickly back to steady state after the shock. This contributes to the rising volatility of output with intangible capital depreciation, in Table 5.

When intangibles display lower rates of depreciation, the peak in TFP occurs later as entrepreneurs reallocate more physical investment to the unmeasured intangible investment sector. Thus final goods fall more upon impact and rise more smoothly over time. This generates a more negative skew in the response of measured TFP in Figure 4, pushing the TFP-to-shock correlation to more negative in Table 5.

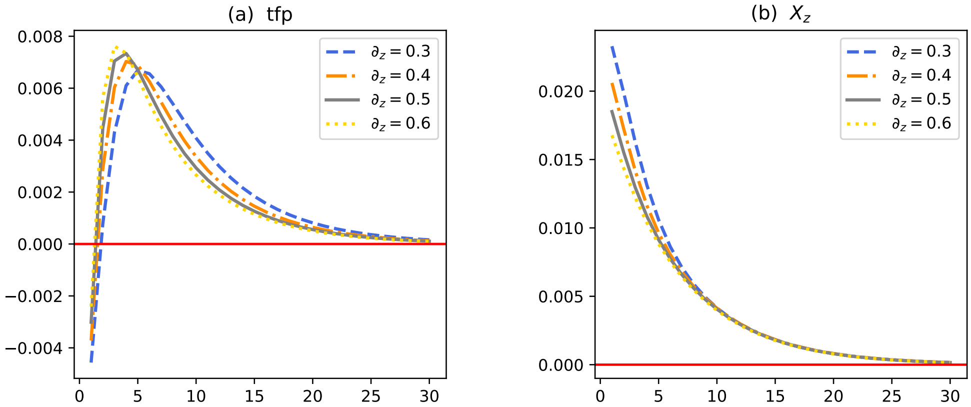

Figure 4: Aggregate TFP and intangible investment's response to a one standard deviation shock to intangible investment for different values of intangible capital depreciation, δ_z. The responses are percent deviations from steady state.

6 Conclusion

We study the role of an intangible investment technology shock in driving and propagating business cycles. We argue that reallocation of physical investment plays a key role in generating the observed comovements in aggregate output, aggregate consumption, aggregate investment and aggregate hours. Such reallocation arises when entrepreneurs who own the firms and the physical capital stock, display a degree of risk aversion that lies within a range that is neither too high nor too low. We use firm level intangible capital data computed by Ewens et al. (2019) for US firms (and other aggregate US series) to calibrate the model and show that the nested cases of both risk neutrality and log-utility does not generate aggregate comovement, particularly between consumption and investment but also between consumption and hours in case of the latter. The model gives rise to plausible business cycles for values of the entrepreneurial risk aversion parameter lying between 0 and 0.75. Borrowing constraints, by making consumption smoothing by entrepreneurs more difficult, further strengthens the inter-sectoral reallocation of physical investment and hence the model's results, although the results of the model go throughout with non-binding financial constraints as well.

We also show that the intangible investment technology shock, although a sectoral productivity shock by definition, has an almost zero correlation with measured aggregate TFP.

This is in keeping with Angeletos et al (2020), who find that a single main shock, while consistent with data on business cycles' drivers, is essentially orthogonal to movements in aggregate TFP at all frequencies. The unmeasured nature of intangible investments in our model renders a shock to these investments uncorrelated to aggregate measured TFP and therefore qualifies as a candidate source for business cycle fluctuations.

References

Angeletos, G.-M., F. Collard, and H. Dellas (2020): "Business-Cycle Anatomy," *American Economic Review*, 110, 3030–70.

Aoki, K. and K. Nikolov (2015): "Bubbles, banks and financial stability," *Journal of Monetary Economics*, 74, 33–51.

Bachmann, R. and C. Bayer (2014): "Investment Dispersion and the Business Cycle," *American Economic Review*, 104, 1392–1416.

Becard, Y. and D. Gauthier (2022): "Collateral Shocks," *American Economic Journal: Macroeconomics*, 14, 83–103.

Bijnens, G. and J. Konings (2020): "Declining business dynamism in Belgium," *Small Business Economics*, 54, 1201–1239.

Buera, F. J. and B. Moll (2015): "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity," *American Economic Journal: Macroeconomics*, 7, 1–42.

Carvalho, V. M. and B. Grassi (2019): "Large Firm Dynamics and the Business Cycle," *American Economic Review*, 109, 1375–1425.

Chahrour, R., K. Nimark, and S. Pitschner (2021): "Sectoral Media Focus and Aggregate Fluctuations," *American Economic Review*, 111, 3872–3922.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104, 27–65.

Corrado, C., C. Hulten, and D. Sichel (2009): "INTANGIBLE CAPITAL AND U.S. ECONOMIC GROWTH," *Review of Income and Wealth*, 55, 661–685.

Crouzet, N. and J. Eberly (2021): "Intangibles, markups, and the measurement of productivity growth," *Journal of Monetary Economics*, 124, S92–S109, the Real Interest Rate and the MarginalProduct of Capital in the XXIst CenturyOctober 15-16, 2020.

Crouzet, N., J. C. Eberly, A. L. Eisfeldt, and D. Papanikolaou (2022): "The Economics of Intangible Capital," *Journal of Economic Perspectives*, 36, 29–52.

De Ridder, M. (2024): "Market Power and Innovation in the Intangible Economy," *American Economic Review*, 114, 199–251.

Ewens, M., R. H. Peters, and S. Wang (2019): "Measuring Intangible Capital with Market Prices," Working Paper 25960, National Bureau of Economic Research.

Gabaix, X. (2011): "The granular origins of aggregate fluctuations," *Econometrica*, 79.

Galí, J. and L. Gambetti (2009): “On the Sources of the Great Moderation,” *American Economic Journal: Macroeconomics*, 1, 26–57.

Gertler, M. and N. Kiyotaki (2010): “Chapter 11 - Financial Intermediation and Credit Policy in Business Cycle Analysis,” Elsevier, vol. 3 of *Handbook of Monetary Economics*, 547–599.

Gourio, F. and L. Rudanko (2014): “Can Intangible Capital Explain Cyclical Movements in the Labor Wedge?” *American Economic Review*, 104, 183–88.

Haskel, J. and S. Westlake (2018): *Capitalism without Capital: The Rise of the Intangible Economy*, Princeton University Press.

Hayashi, F. (1985): “Corporate finance side of the Q theory of investment,” *Journal of Public Economics*, 27, 261–280.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): “Investment shocks and business cycles,” *Journal of Monetary Economics*, 57, 132–145.

Kehrig, M. and N. Vincent (2019): “Good Dispersion, Bad Dispersion,” Working Paper 25923, National Bureau of Economic Research.

Koh, D., R. Santaeulàlia-Llopis, and Y. Zheng (2020): “Labor Share Decline and Intellectual Property Products Capital,” *Econometrica*, 88, 2609–2628.

McGrattan, E. R. and E. C. Prescott (2010): “Unmeasured Investment and the Puzzling US Boom in the 1990s,” *American Economic Journal: Macroeconomics*, 2, 88–123.

——— (2014): “A Reassessment of Real Business Cycle Theory,” *American Economic Review*, 104, 177–82.

Mitra, S. (2019): “Intangible capital and the rise in wage and hours volatility,” *Journal of Economic Dynamics and Control*, 100, 70–85.

Peters, R. H. and L. A. Taylor (2017): “Intangible capital and the investment-q relation,” *Journal of Financial Economics*, 123, 251–272.

Prescott, E. and E. McGrattan (2012): “The Labor Productivity Puzzle,” 2012 Meeting Papers 644, Society for Economic Dynamics.

Stock, J. H. and M. W. Watson (2002): “Has the Business Cycle Changed and Why?” Working Paper 9127, National Bureau of Economic Research.

van Vlokhoven, H. (2024): “Estimating the Cost of Capital and the Profit Share*,” *The Economic Journal*, ueae001.