

Return Risk Measurement: Orlicz-Type Measures of Risk

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Abstract

In this work we provide an axiomatic foundation of Orlicz measures of risk in terms of properties of their acceptance sets, by exploiting their natural correspondence with shortfall risk (Föllmer and Schied, 2004). We explicate that, contrary to common use of *monetary risk measures*, which measures the risk of a financial position by assessing the stochastic nature of its *monetary* value, Orlicz measures of risk assess the stochastic nature of *returns*: they are *return risk measures*.

This axiomatic foundation of Orlicz measures of risk naturally leads to several *robust* generalizations, obtained by generalizing expected utility to ambiguity averse preferences such as variational preferences (Maccheroni et al., 2006) and homothetic preferences (Cerreia-Vioglio et al., 2011, Chateauneuf and Faro, 2010, Laeven and Stajic, 2013). We also consider the case of ambiguity (multiplicity) over the Young function Φ in the definition of the Orlicz measure of risk and the case of a state-dependent Φ leading to *Musiak-Orlicz measures of risk*. From a purely mathematical point of view, the resulting functionals can in a unified way be seen as suprema of Orlicz norms on a suitable rearrangement-invariant Banach space.

We study the properties of these *robust Orlicz measures of risk* and analyze and provide dual representations of their optimized translation invariant extensions (Rockafellar and Uryasev, 2000, Rockafellar, Uryasev and Zabarankin, 2008), that generalize the class of Haezendonck-Goovaerts risk measures, leading to *robust Haezendonck-Goovaerts measures of risk*. An application to optimal risk sharing is also provided.

Keywords: Return risk measures; Shortfall risk; Robustness; Ambiguity averse preferences; Orlicz norms and spaces; Convex risk measures; Positive homogeneity.

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