# Polynomial approximations for multivariate aggregate claims amount probability distributions

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**Abstract**

A numerical method to compute multivariate probability distributions from their Laplace transform is presented. The method consists in an orthogonal projection of the probability density function with respect to a probability measure that belongs to Natural Exponential Family with Quadratic Variance Function (NEF-QVF). The procedure allows a quick and accurate calculation of probabilities of interest and does not require strong coding skills. Numerical illustrations and comparisons with other methods are provided. This work is motivated by actuarial applications. We aim at recovering the joint distribution of two aggregate claims amount associated with two insurance policy portfolios that are closely related, and at computing survival functions for reinsurance losses in presence of two non-proportional reinsurance treaties. The multivariate aggregate claims amount model is

$$\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = \sum_{j=1}^{N} \begin{pmatrix} U_{1j} \\ \vdots \\ U_{nj} \end{pmatrix} + \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix},$$

where $N$ is a counting random variable that models the number of claims over a given period of time, $\{\mathbf{U}_j\}_{j\in\mathbb{N}}$ is a sequence of random vector **i.i.d.** from a multivariate probability distribution $\mathbb{P}_{\mathbf{U}}$ and $\mathbf{Y} = (Y_1, \dots, Y_n)$ is a random vector whose components are independant. The random variable $N$, the random vector $\mathbf{Y}$ and the random vector sequence $\{\mathbf{U}_j\}_{j\in\mathbb{N}}$ are assumed to be independant. We suppose that $\mathbf{X}$ represents aggregate claim amounts of the same line of business for different insurance companies over a given time period. A reinsurer is proposing a stop-los reinsurance treaty to each insurance company with $\max\left[(X_i - c_i)_+, b_i\right]$, where $i = 1, \dots, n$, $c_i$ denotes the priority and $b_i$ the limit for the insurer $i$. We need the joint distribution of $\mathbf{X}$ to study, for instance, the total cost of reisurance

$$Z = \sum_{i=1}^{n} \max\left[(X_i - c_i)_+, b_i\right].$$

*Keywords:* Multivariate aggregate claims model, multivariate distribution, multivariate Laplace Transform, numerical inversion of Laplace transform, natural exponential families with quadratic variance functions, orthogonal polynomials.