Optimal Consumption-Life Insurance Rules in Incomplete Market

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The purpose of this paper is to apply the good-deal pricing bound theory, proposed by Cochrane and Saa-Requejo (2000), to a continuous-time consumption/life-insurance purchases dynamic decision problem in an incomplete market. Previous studies of optimal consumption/life-insurance purchases decision have some drawbacks: (1) Previous studies (e.g., Richard (1975), Pliska and Ye (2007), Zhu (2007), Ye (2008)) almost assume that the life insurance market is complete market. However, market incompleteness arises because that the mortality risk is non-diversifiable. (2) Previous studies (e.g., Richard (1975), Pliska and Ye (2007), Zhu (2007), Ye (2008)) use the hazard function to represent the instantaneous death rate for the insured and assume that the hazard function is deterministic.

In this paper, the insured's mortality shock is described using a first- passage time model with geometric Brownian motion. Market incompleteness arises from the non-diversifiable riskiness on the non-tradable stochastic process of the insured's health status. Such a type of riskiness is non-hedgeable, and hence, makes the failure of the single-price law as well as no-arbitrage condition. Based on the model setup, we derive the joint probability density of death and health status, and in contrast to prior literature which exogenously determines the premium rate of life insurance, we solve the implied annualized premium rate. Furthermore, we derive the closed-form solution for optimal consumption-life insurance rule by using the dynamic programming approach. In numerical analysis, we discuss the characteristics of health-status dynamics, stochastic discount factor on the insurance-income ratio in a complete and an incomplete market, respectively. We employ empirical analysis to investigate whether the impact of health-status dynamics, stochastic discount factor on the insurance-income ratio is consistent with empirical result. We note that this research is a pioneer in the closed-form solution for optimal consumption-life insurance rule by using the dynamic programming approach in an incomplete market.

Keywords: Optimal consumption-life insurance rule, incomplete market, first-passage time model, dynamic programming approach

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