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**TWO-STEPS MIXED PENSION SYSTEM:  
THE CASE OF THE REPRESENTATIVE INDIVIDUAL  
WITHIN A SOCIAL SECURITY NOTIONAL DEFINED  
CONTRIBUTION FRAMEWORK**

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# 1. How to cope with ageing

## RISK OF THE PAY AS YOU GO SYSTEM

- Fertility rate far below the replacement level
- Increasing life expectancy
- Unemployment



A person of 65 years  
is really an old person ??????

| Equivalent age today to 65 in 1900 (*) |   |              |  |              |
|--|---|--------------|--|--------------|
|  | Age at which 26.2% of a generation survives in... |              | Age at which age expectancy is 9.1 years in... |              |
|  | 1900  | 2015         | 1900   | 2015         |
| Men                                    | 65 years old                                      | 89 years old | 65 years old                                   | 79 years old |
| Women                                  | 65 years old                                      | 93 years old | 65 years old                                   | 82 years old |
| All                                    | 65 years old                                      | 91 years old | 65 years old                                   | 81 years old |

(\*) In 1900, at 65 years old, a 26.2% of a generation survived in Spain and remaining life was 9.1 years (unisex)

Fuente: own computations from data from INE

## 1. How to cope with ageing

### ONE FIRST SOLUTION: COMPLEMENTARY SYSTEM !!!!!

## BUT ANNUITIES

- Mitchell et al. (1999) show that lifetime annuities in the United States are between 15% and 25% lower than those obtained when using overall population life tables.
- Finkelstein and Poterba (2000) also show that lifetime annuities in the UK, which are taken out by 65-year-old men, are between 10% and 15% lower than those that would be obtained using ordinary overall population life tables.
- Domínguez-Fabian, (2017) find that in Spain annuities may be also around 29% lower than those obtained using ordinary life tables

# 1. How to cope with ageing

## Overloaded annuities: The Spanish case

**Table 1. The impact of longevity risk on annuities (term versus life) using Spanish Mortality Tables**

| Hypothesis of analysis   |                  |                  |
|--|------------------|------------------|
| Starting age of contributions (with continuous contributions during the entire working life) | 30               |                  |
| Retirement age   | 67               |                  |
| Grand age (a)  | 78               |                  |
| Salary at the starting age of contributions  | 20,000 €         |                  |
| Interest rate  | 3%               |                  |
| Annual increase in wages   | 3%               |                  |
| Expected inflation   | 2%               |                  |
| Results  | Term annuity (b) | Life annuity (c) |
| Annuity computed using PERM/F 2000 Table   | 33,593.76 €      | 14,359.05 €      |
| Annuity computed using the ordinary Mortality Table  | 35,289.29 €      | 20,299.95 €      |
| Annuity value gap due to compensation for longevity  | -4.80%           | -29.27%          |

Notes:

- (a) Age at which the Social Security initiates the payment of pensions in the two-steps mixed system (see below)
- (b) Term annuity individuals obtain between the retirement age and the grand age, when the SS grants their life annuities
- (c) Life annuity at retirement age

Source: Compiled from Herce and del Olmo (2013)

Source: Domínguez-Fabián, (2017)

- **The problem that occurs in the case of lifetime annuities (because of adverse selection and over-weighted mortality) is significantly reduced in the case of temporary annuities (even if still present)**

# 1. How to cope with ageing

## PROBLEMS

1. The Contributors **do not** want to retire later.
2. The Contributors **obtain a low return from their** contributions to the **private saving with the annuity.**

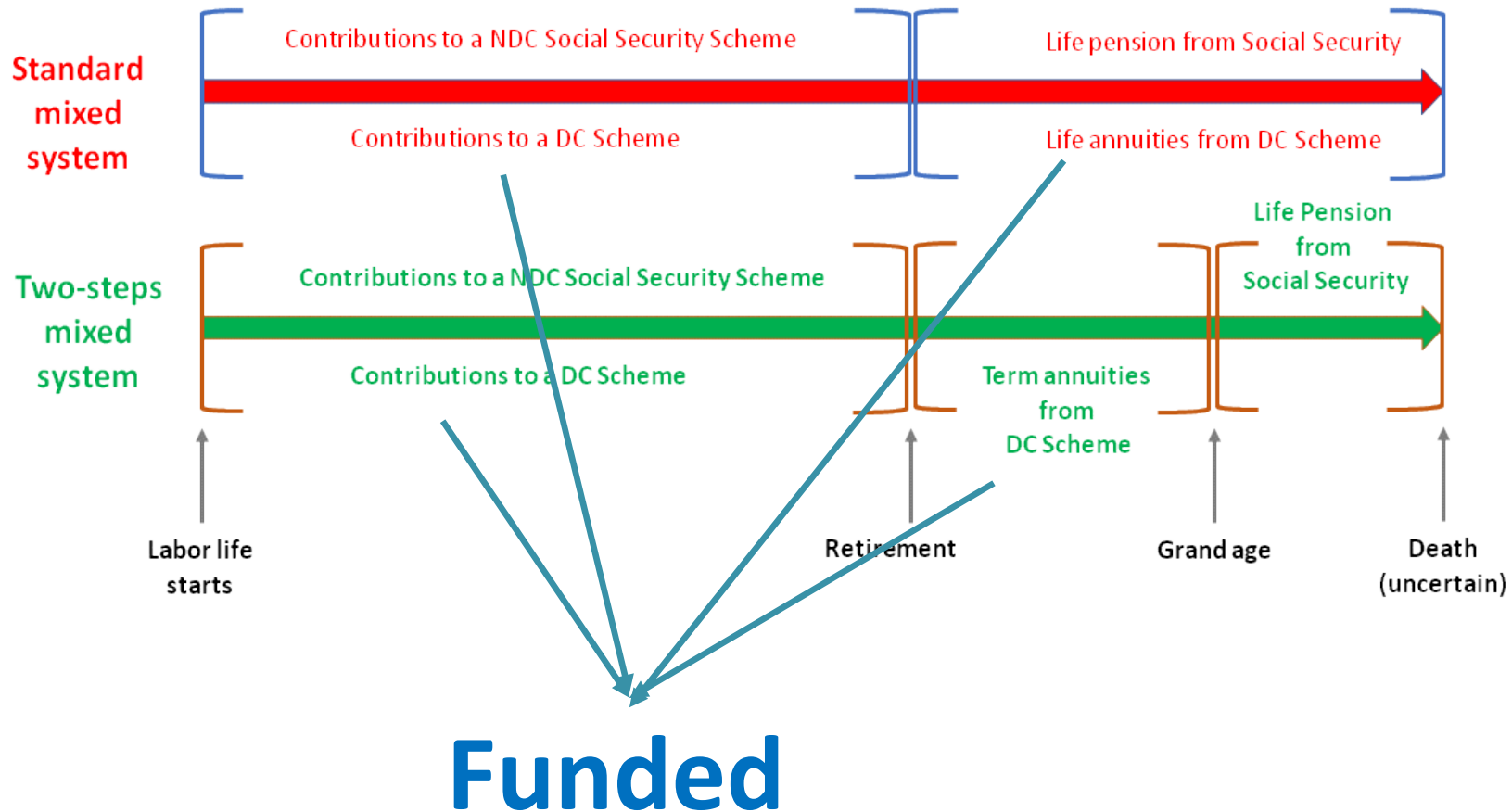


3. The Social Security has problem to pay the same pensions during more time.
4. The Social Security has problem because **people do not** want to retire later.
5. The Social Security has problem with the balance: deficit

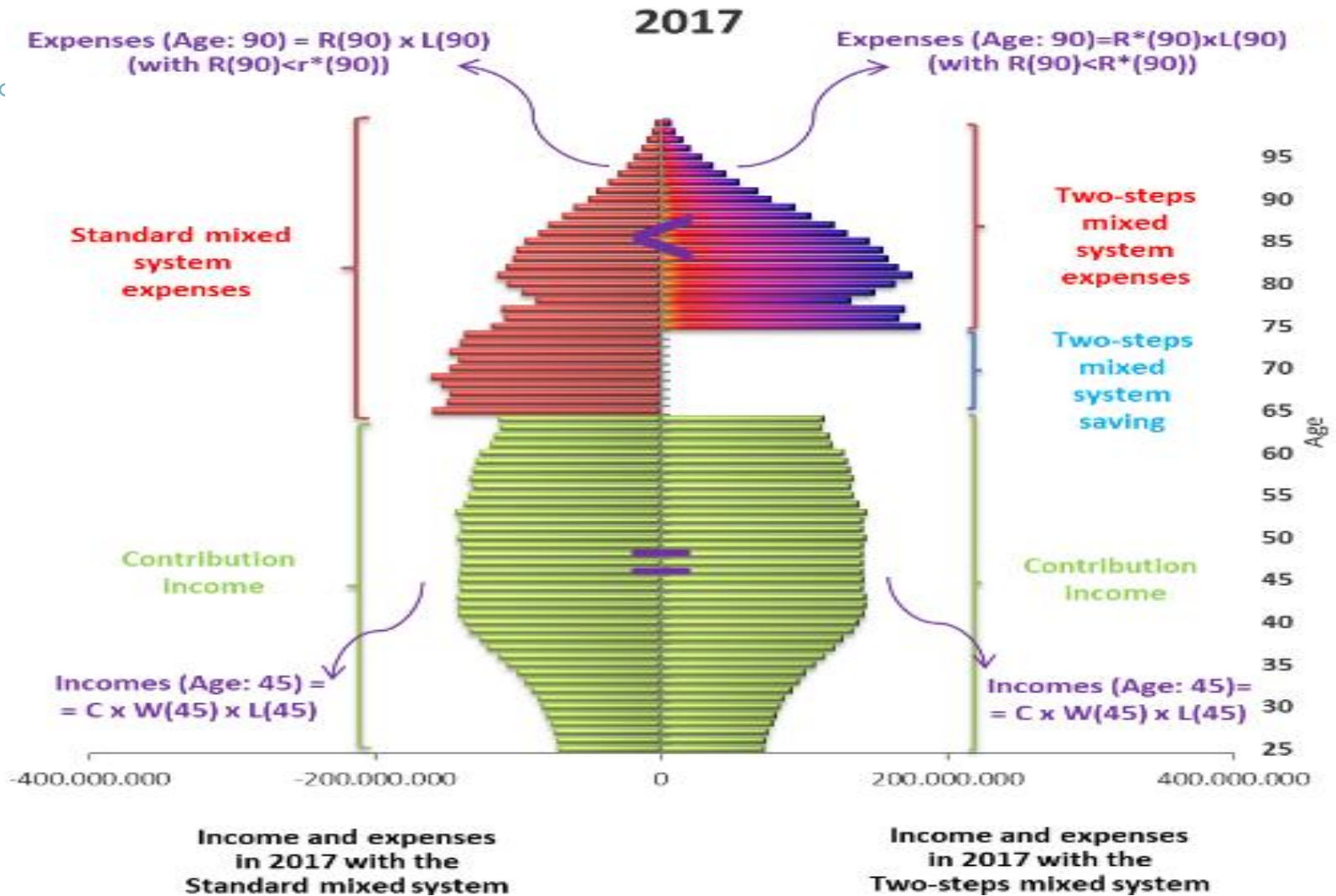
<http://celebracnc.com/2017/08/28/el-poder-para-solucionar-problemas/>

6. The insurers have problems because a lot of **people “decide” not to save** in social prevision

## 2. A Two-steps Mixed Pension System

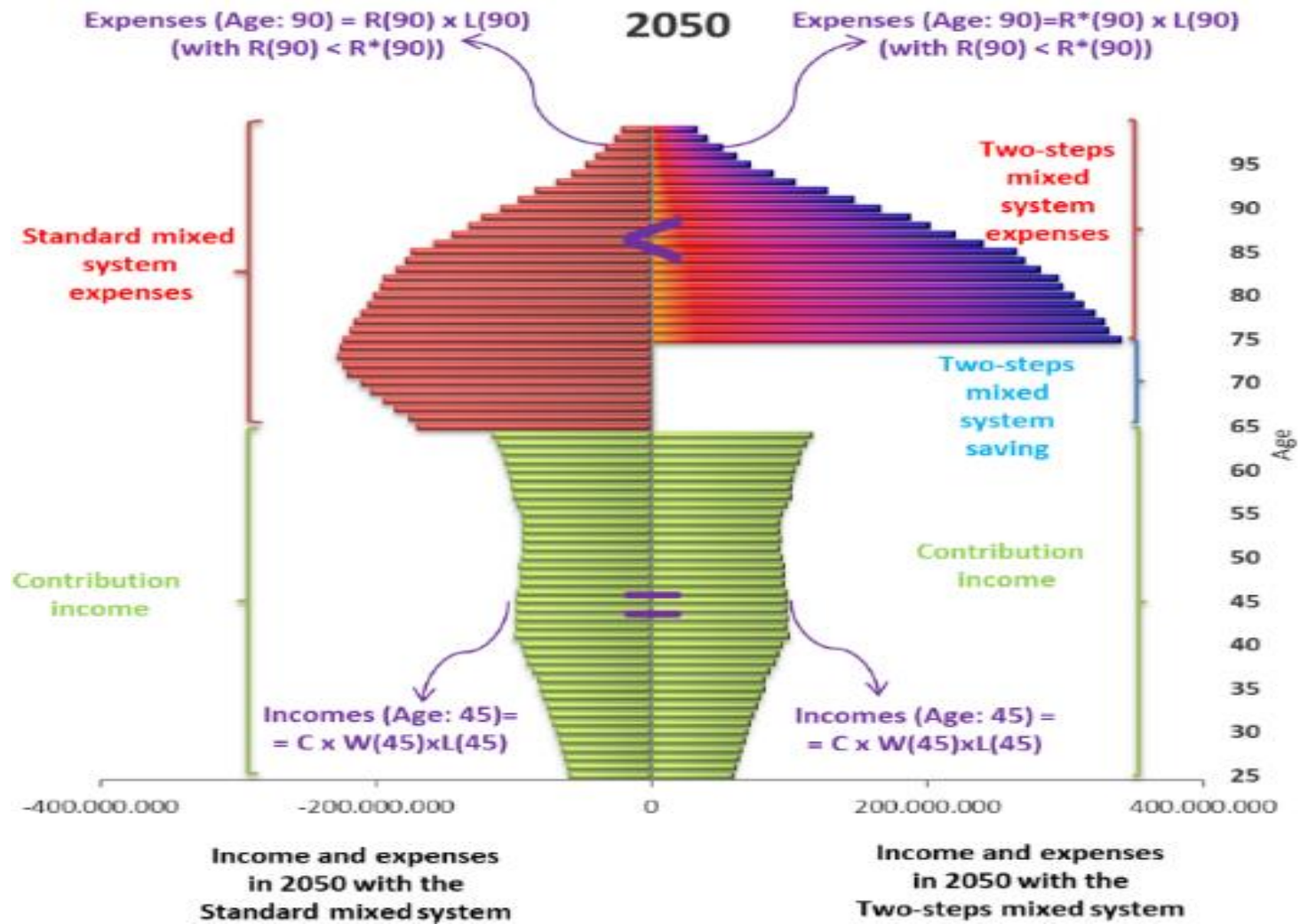


## 2. A Two-steps Mixed Pension System





## 2. A Two-steps Mixed Pension System



## 2. A Two-steps Mixed Pension System

### Modelling

#### Assumptions :

Social Security system in DC with a part in **PAYG** ( NDC) and a part in **funding**

$\pi_N$  = fixed contribution rate in NDC

$\pi_F$  = fixed contribution rate in funding

**Accumulation** period between age  $x_0$  and age  $x_r$  (  $N = x_r - x_0$  )

**Decumulation** period between age  $x_r$  and age  $w$

Comparison between the standard model and the 2-steps system from an **individual** point of view ( *level of the benefits through the IRR* ) and an **aggregated** point of view ( *balance sheet of the social security PAYG system* )

## 2. A Two-steps Mixed Pension System

### Accumulation period in funding

- Funded account at retirement age ( financial computation without mortality) :

$$C_F = \pi_F \cdot \left( \sum_{j=0}^{x_r-1-x_0} (S(0)(1+\beta)^j(1+k)^j) \cdot (1+i)^{N-j} \right)$$

Initial wage      Inflation      Scale      Financial return

$$C_F = \pi_F \cdot S(0) \cdot (1+i) \cdot \frac{(1+i)^N - (1+\beta)^N \cdot (1+k)^N}{(1+i) - (1+\beta) \cdot (1+k)}$$

It is the same form with the two models!

## 2. A Two-steps Mixed Pension System

### Accumulation period in NDC

- Notional account at retirement age ( notional computation without mortality) :

$$C_N = \pi_N \cdot \left( \sum_{j=0}^{x_r-1-x_0} (S(0)(1+\beta)^j(1+k)^j) \cdot (1+r)^{N-j} \right)$$

Initial wage      Inflation      Scale      Notional rate

Canonical choice :  $1+r = (1+\beta) \cdot (1+d)$       Demographic increase

$$C_N = \pi_N \cdot S(0) \cdot (1+r) \cdot \frac{(1+r)^N - (1+\beta)^N \cdot (1+k)^N}{(1+r) - (1+\beta) \cdot (1+k)}$$

It is the same form with the two models!

## 2. A Two-steps Mixed Pension System

### Decumulation period in the standard model *Funded part*

Conversion of the funded capital into a **life annuity** using an insurer tariff ( safety margin on the survival probabilities ; management costs )

$P_F(x_r)$  = initial pension at retirement age for the funded part

$$P_F(x_r) = \frac{C_F}{a_{x_r}^F}$$

$$a_{x_r}^F = \frac{1}{1-g} \cdot \left( \sum_{x=x_r}^{\omega} \bar{p}_{x_r} \cdot (1+\beta)^{x-x_r} / (1+i)^{x-x_r} \right)$$

Management  
cost

Insurer life table  
for lifetime annuity

## 2. A Two-steps Mixed Pension System

### Decumulation period in the standard model *NDC part*

Conversion of the notional account into a *life annuity*

$P_N(x_r)$  = initial pension at retirement age for the notional part

$$P_N(x_r) = \frac{C_N}{a_{x_r}^N}$$

$$a_{x_r}^N = \left( \sum_{x=x_r}^{\omega} p_{x_r} \cdot (1+\beta)^{x-x_r} / (1+r)^{x-x_r} \right)$$

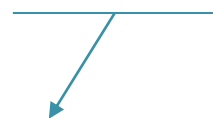
Population life table

## 2. A Two-steps Mixed Pension System

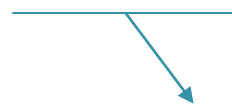
### Decumulation period in the **standard** model *Total*

Pension at age x :  $x_r \leq x < \omega$

$$P(x) = P_F(x_r).(1 + \beta)^{x-x_r} + P_N(x_r).(1 + \beta)^{x-x_r}$$



Funded  
Part



Notional  
Part

## 2. A Two-steps Mixed Pension System

### Decumulation period in the 2-steps model *Funded part*

Conversion of the funded capital into a **term annuity** using an insurer tariff ( safety margin on the survival probabilities ; management costs ) between retirement age  $x_r$  and great age  $y$

$P_F(x_r)$  = initial pension at retirement age for the funded part

$$P_F(x_r) = \frac{C_F}{a_{x_r, y-x_r}^F}$$

$$a_{x_r, y-x_r}^F = \frac{1}{1-g} \cdot \left( \sum_{x=x_r}^{y-1} \beta_{x_r}^{\%} \cdot (1+\beta)^{x-x_r} / (1+i)^{x-x_r} \right)$$

Management  
cost

Insurer life table  
for temporary annuity



Insurer life table  
for lifetime annuity



## 2. A Two-steps Mixed Pension System

*NDC part*

### Decumulation period in the 2-steps model

Conversion of the notional account into a **life annuity** starting at the great age  $y$  (**deferred annuity**)

$P_N(y)$  = initial pension at great age for the notional part

$$P_N(y) = C_N \cdot \frac{(1+r)^{y-x_r} \cdot (1-\delta) / {}_{y-x_r}P_{x_r}}{a_y^N}$$

$$a_y^N = \left( \sum_{x=x_r}^{\omega} {}_{x-x_r}P_{x_r} \cdot (1+\beta)^{x-x_r} / (1+r)^{x-x_r} \right)$$

## 2. A Two-steps Mixed Pension System

### Decumulation period in the 2-steps model *Total*

Pension at age x :

$$x_r \leq x < y$$

$$P(x) = P_F(x_r) \cdot (1 + \beta)^{x-x_r}$$

Funded Part

FIRST STEP

$$y \leq x < \omega$$

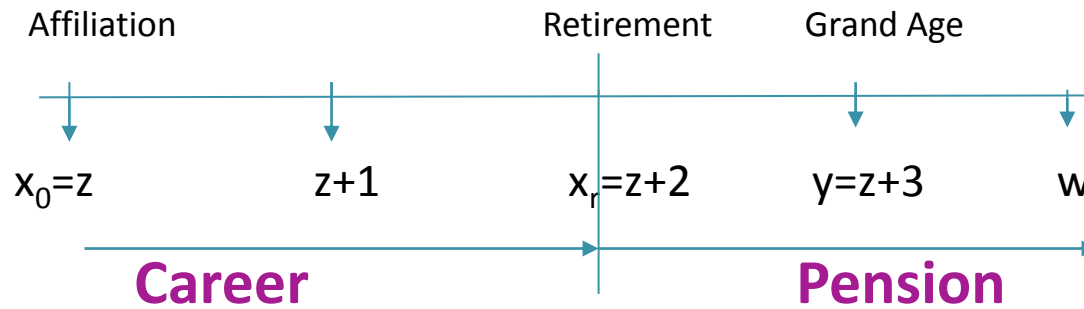
$$P(x) = P_N(y) \cdot (1 + \beta)^{x-y}$$

Notional Part

SECOND STEP

### 3. Individual perspective: IRR

#### Model over 4 periods of time



| Hypothesis  |                   |        |
|-------------|-------------------|--------|
| FUNDING     | Contribution rate | 0.04   |
|             | Funding rate      | 0.04   |
|             | Management cost   | 0.0125 |
| NDC         | Contribution rate | 0.16   |
|             | Notional rate     | 0.04   |
| Inflation   | 0.02              |        |
| Probability | 0.95              |        |
| Scale       | 0.01              |        |

Standard Model

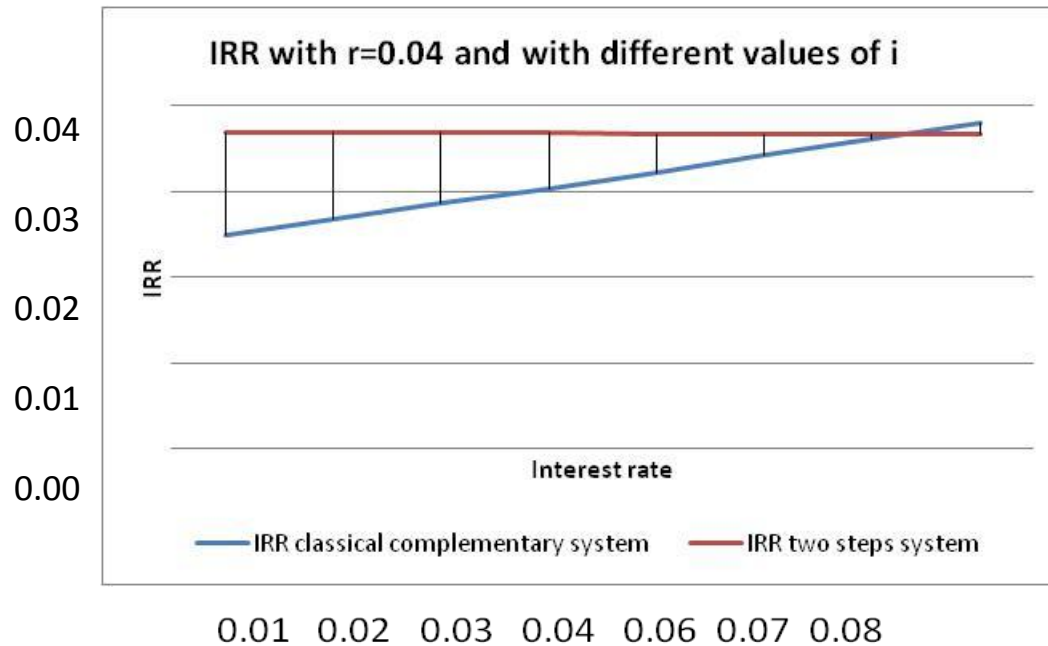
$$\text{IRR} = 0.03038558$$

2-steps Model

$$\text{IRR} = 0.03157632$$

### 3. Individual perspective: IRR

Case 1. Without commission in funding and using the same life table in notional and in funding and with the interest rate  $i$  different from the notional rate  $r$ .



Case 2. It is interesting to work with the realistic case with commission in funding and using the different life table in notional and in funding and with the interest rate  $i$  different from the notional rate  $r$ .



## 4. Aggregate perspective: balance sheet

### Model over 4 periods of time / NDC part

| POPULATION |     |         |         |         |         |
|------------|-----|---------|---------|---------|---------|
|            |     | TIME    |         |         |         |
|            |     | 0       | 1       | 2       | 3       |
| AGE        | Z   | 100,000 | 101,960 | 103,958 | 105,996 |
|            | Z+1 | 98,078  | 100,000 | 101,960 | 103,958 |
|            | Z+2 | 96,192  | 98,078  | 100,000 | 101,960 |
|            | Z+3 | 89,626  | 91,383  | 93,174  | 95,000  |

| INDIVIDUAL WAGE |     |      |        |        |        |
|-----------------|-----|------|--------|--------|--------|
|                 |     | TIME |        |        |        |
|                 |     | 0    | 1      | 2      | 3      |
| AGE             | Z   | 100  | 102    | 104.04 | 106.12 |
|                 | Z+1 | 101  | 103.02 | 105.08 | 107.18 |

## 4. Aggregate perspective: balance sheet

### Model over 4 periods of time / NDC part

#### Classical model

| BALANCE  |              |              |              |              |
|----------|--------------|--------------|--------------|--------------|
|          | TIME         |              |              |              |
|          | 0            | 1            | 2            | 3            |
| Incomes  | 3,981,169.09 | 4,140,384.00 | 4,305,966.24 | 4,478,170.44 |
| Outcomes | 3,981,169.09 | 4,140,384.00 | 4,305,966.24 | 4,478,170.44 |
| Balance  | 0            | 0            | 0            | 0            |

#### Two steps model



| BALANCE  |                         |                         |                         |                         |
|----------|-------------------------|-------------------------|-------------------------|-------------------------|
|          | TIME                    |                         |                         |                         |
|          | 0                       | 1                       | 2                       | 3                       |
| Incomes  | 3,981,169.09            | 4,140,384.00            | 4,305,966.24            | 4,478,170.44            |
| Outcomes | 3,961,263.24            | 4,119,682.08            | 4,284,436.41            | 4,455,779.59            |
| Balance  | <b><u>19,905.85</u></b> | <b><u>20,701.92</u></b> | <b><u>21,529.83</u></b> | <b><u>22,390.85</u></b> |



## 5. Concluding comments

Possible to calibrate the **2 steps** system to obtain added value for all the stakeholders

- Better IRR for the *Contributors*
- Better balance for the *social security*
- Better design of product for *insurers*



<https://fr.dreamstime.com/images-libres-de-droits-r%C3%A9solution-des-probl%C3%A8mes-image38483749>



# *Thank you*

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